BASIC ARRAY PROCESSING CONCEPTS (from „shift-and-sum“ to „frequency wavenumber spectra“) and related things...

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with contributions by Cécile Cornou, Marc Wathelet
and the SESAME partners

various illustrations taken from textbook
”Array signal processing – concepts and techniques”
by Don H. Johnson and Dan E. Dudgeon
Prentice Hall Sig. Proc. Series

OVERVIEW: Basic Array Method Principles

- Definition: what is a seismic array?
- What are the benefits of seismic arrays?
- Seismic arrays: historical context and developments
- Basic assumption for array processing:
  the need for a wave propagation model
- Plane wave parameter determination
- Delay-and-sum beamforming
- Frequency-wavenumber spectrum

OVERVIEW: Array Geometry → Limitations

- Parameters used to describe array geometries
- Starting simple: 1D layouts
- Relation of parameters with array behaviour
- Discrete sampling of wavefield and implications
- Generalization to planar 2D-geometries
- Directional dependence of array behaviour
- The quest for an optimal array geometry – an old and (maybe) endless story...

OVERVIEW: Array Analysis of Microtremor Wavefields
Applying basic principles to a special problem domain

- What is special with microtremor wavefields?
- What is to be changed from the viewpoint of analysis?
- What is to be changed from the viewpoint of geometries?

→ Complications and attempts to deal with them

BASIC ARRAY METHOD PRINCIPLES

Basic Array Method Principles

seismic network!

Definition: what is a ‚seismic array‘ ?

{ set of seismograph stations with common time base

AND

sensors located closely enough in space

so that arriving seismic signal waveforms can be correlated between adjacent sensors

seismic array!

to be defined later

Basic Array Method Principles (general)

We can conclude from the definition:

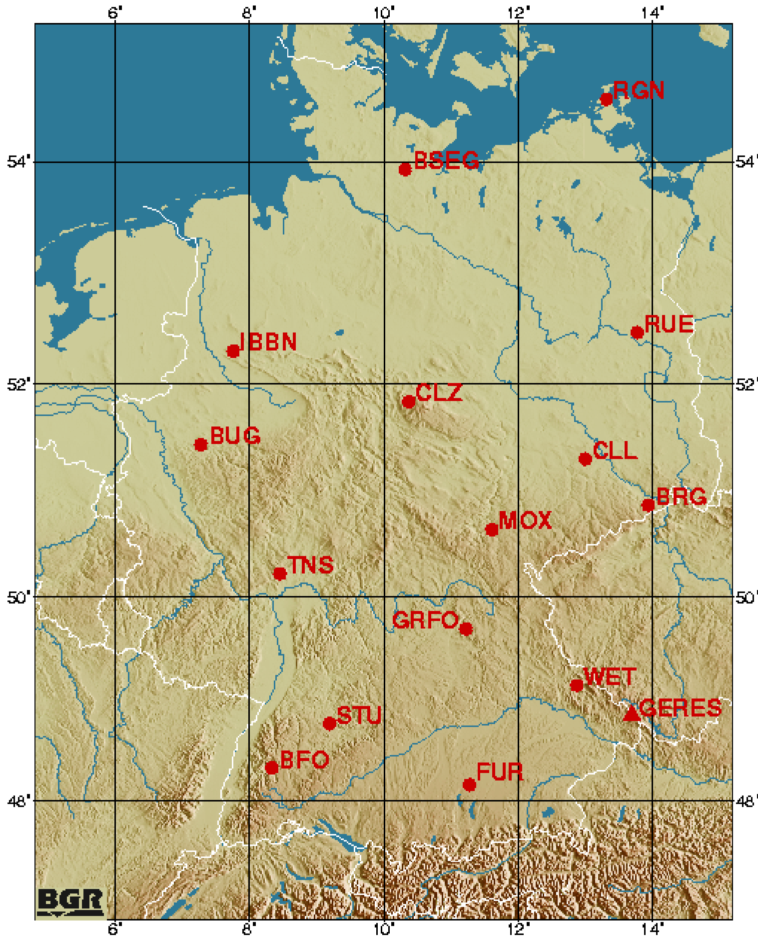
set of seismograph stations with common time base

may act

BOTH as ‚seismic array‘ **AND** ‚seismic network‘

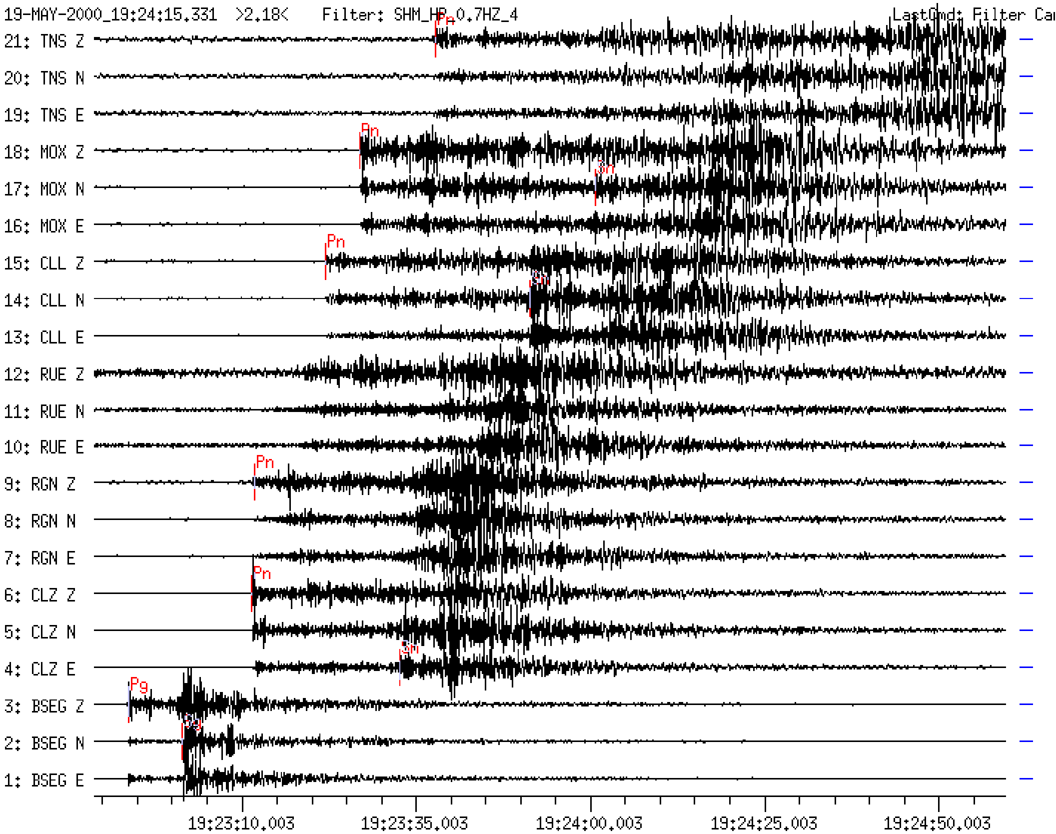
depends on application / wavefield properties of interest

German Regional Seismic Network: array AND network!



German Regional Seismic Network: network operation

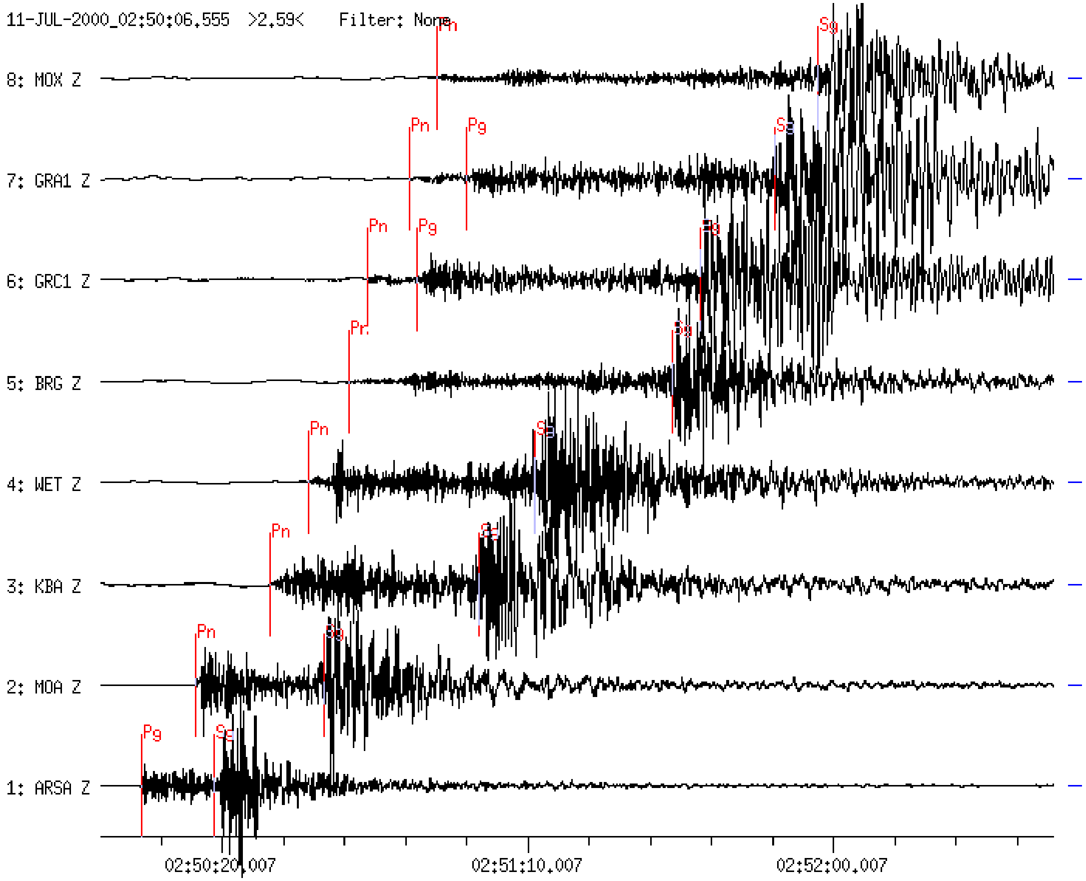
2000-05-19 OT 19:22:40.8(UTC) 53.47N 11.10E Ml 3.4
Wittenburg (W-Mecklenburg). Local earthquake recorded at 7 GRSN 3C-stations.



From http://www.szgrf.bgr.de/seismo_examples.html

German Regional Seismic Network: network operation

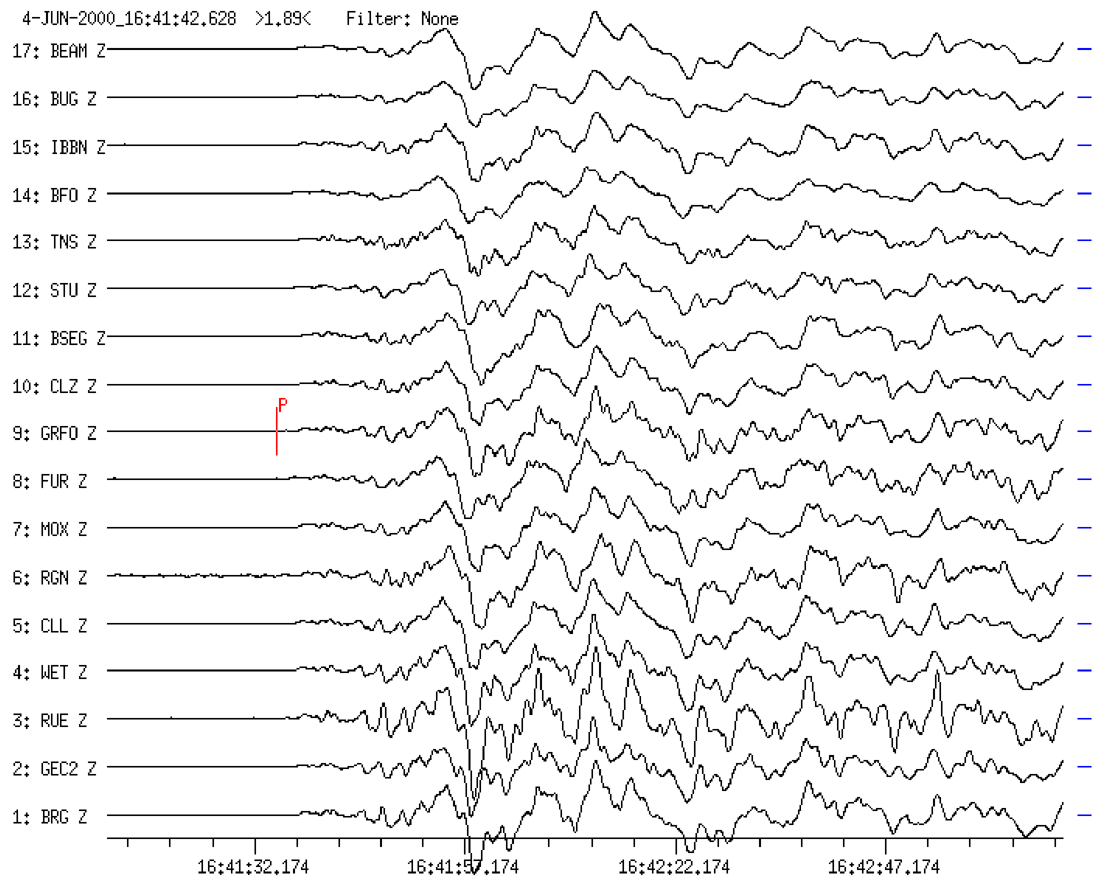
Regional earthquake south of Wien (Ml 5.2)
2000-07-11 OT 2:49:51(UTC) 48.10 N 16.40 E Ml 5.2



http://www.szgrf.bgr.de/seismo_examples.html

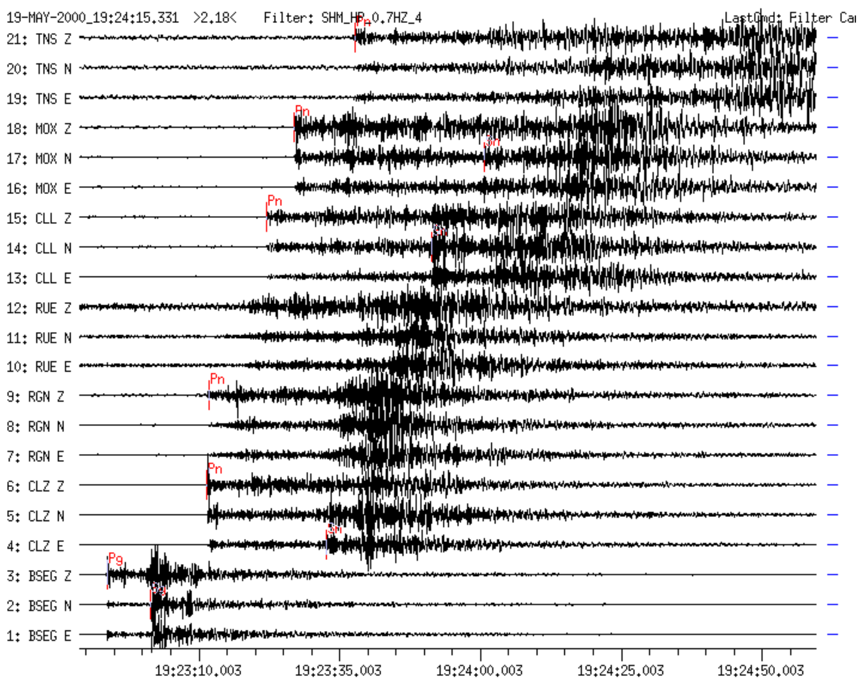
German Regional Seismic Network: array operation

Earthquake in Southern Sumatera Region (distance 94.1° to GRF site, az. 92.5°, depth 33km)
USGS NEIC-data: 2000-06-04 OT 16:28:25.8 4.773 S 102.050 E depth 33km mb 6.8 Ms 8.0

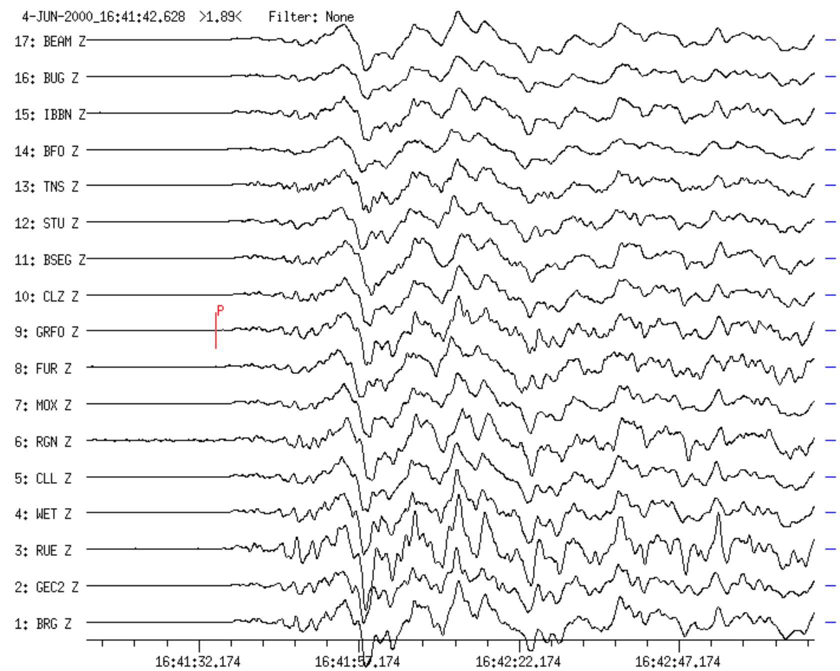


German Regional Seismic Network:

network operation



array operation



Note the difference?

Benefits of seismic arrays

The benefit of seismic arrays can be immediately recognized by considering the information content of seismic observations for various settings:

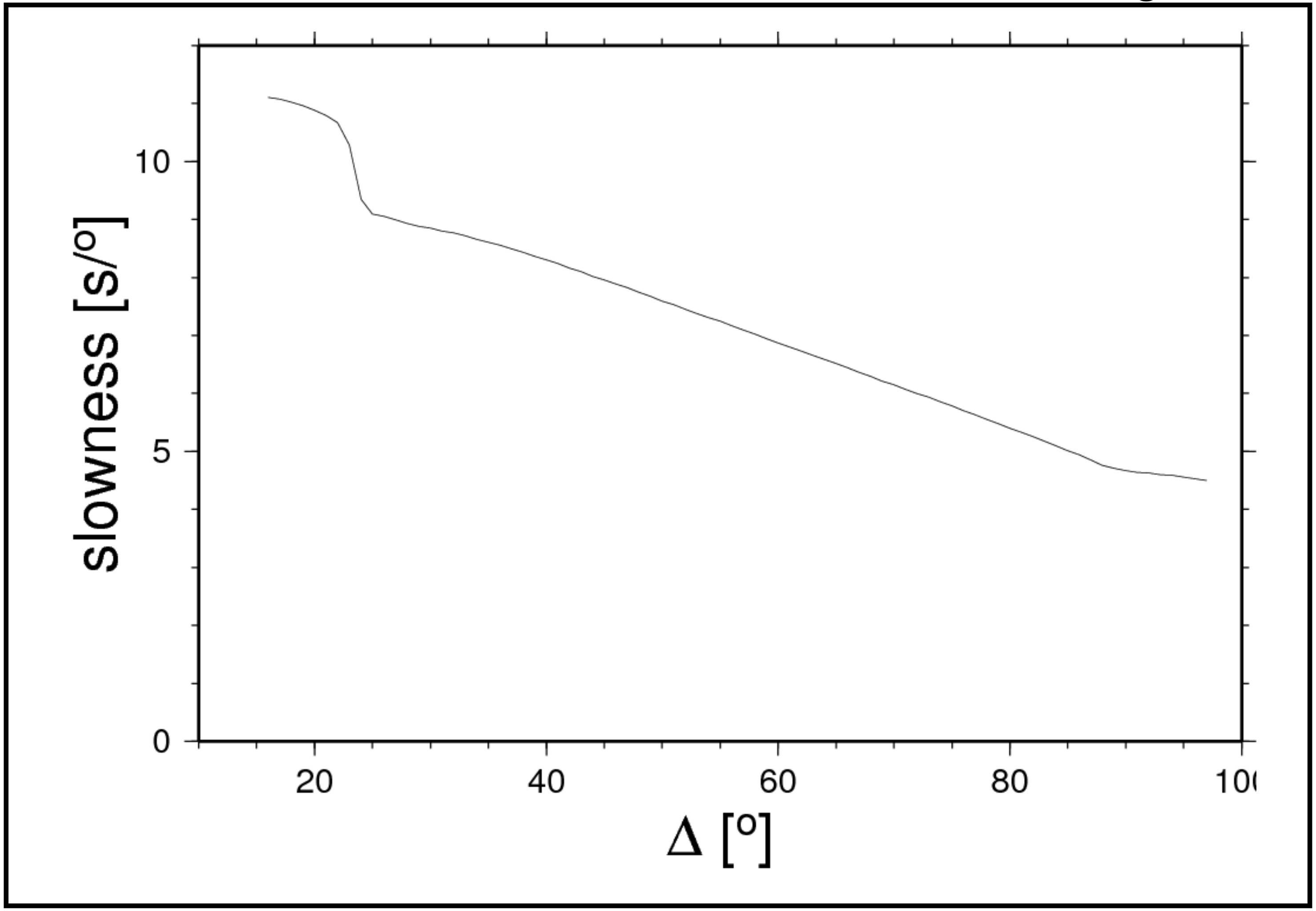
- single station single component
- single station three components
- seismic network (1 or 3 components)
- **seismic array** (1 or 3 components)

available information for:

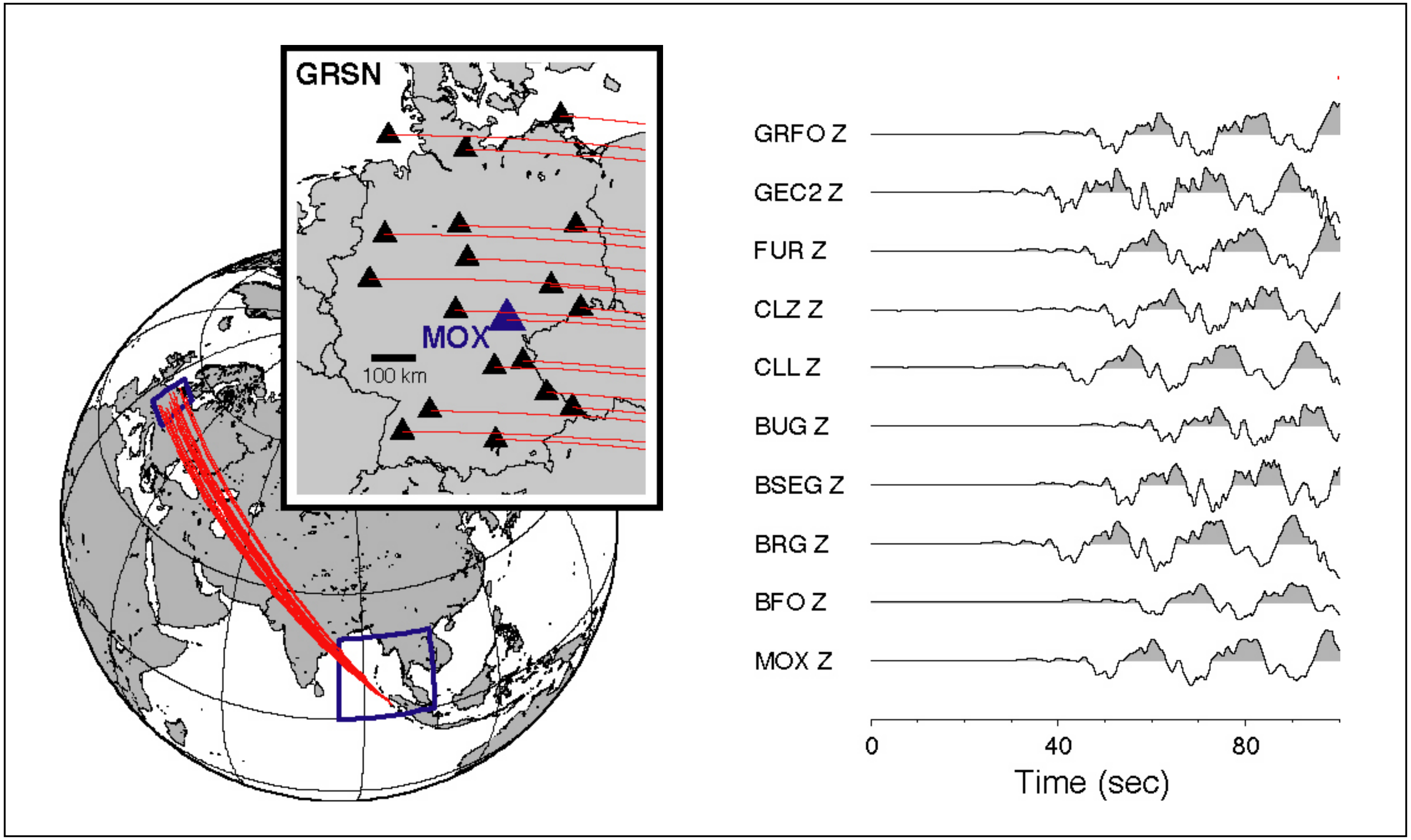
- single station single component:
  arrival times, amplitudes
- single station three components
  arrival times, amplitudes, polarization
  (= particle motion at site)
- seismic network (1 or 3 component)
  arrival times, amplitudes, (polarization),
  direction of wave propagation (indirect from location),
- seismic array (1 or 3 component)
  arrival times, ampl., (polarization), direction of wave, apparent propagation velocity of wave, SNR improvment, eventually wave type (from combination of polarization and wave propagation direction + apparent velocity)

Apparent velocity ←→ horizontal slowness ←→ ray parameter

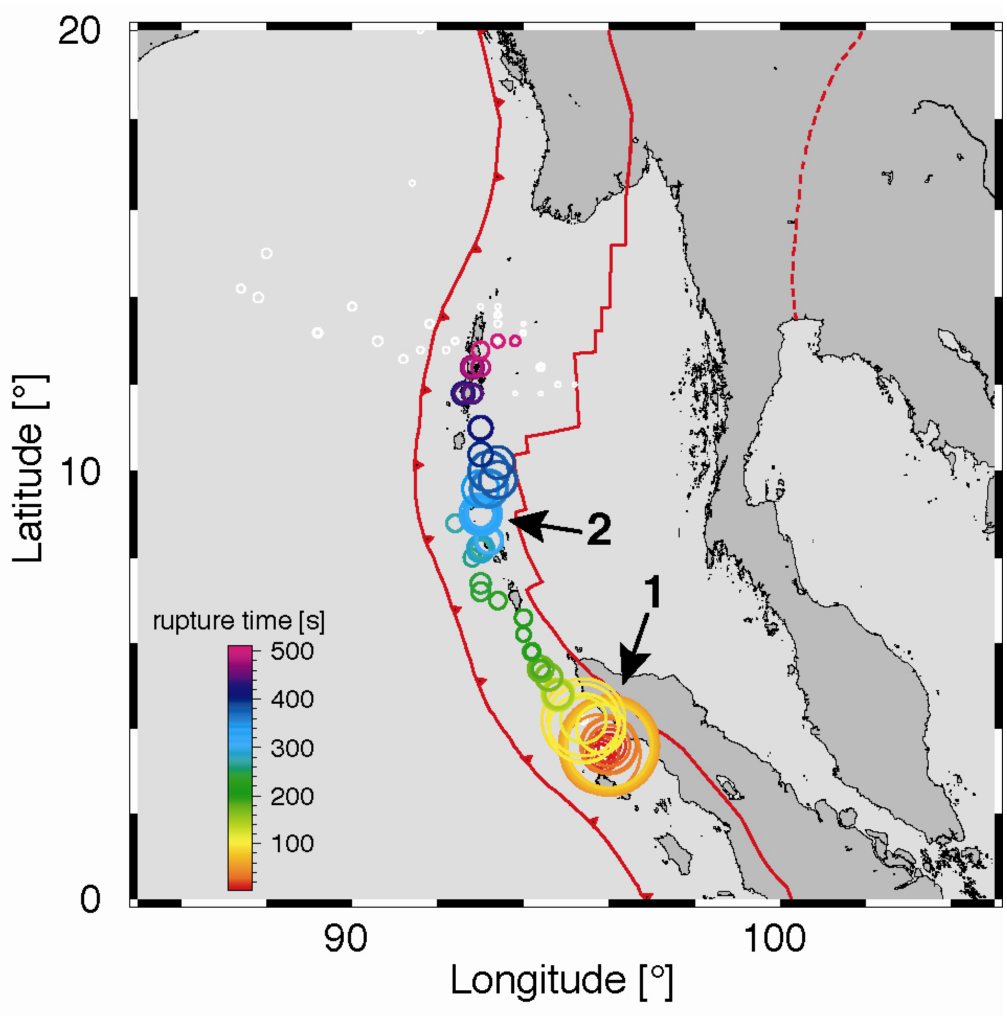
Slowness – distance relation in teleseismic distance range



German Regional Seismic Network: array operation



Using ambient vibration techniques for site characterization
Thessaloniki, 21.02.-28.02.2010



Krüger and Ohrnberger, 2005

Seismic arrays: historical context and developments

(summary from Mykkeltveit et al., 1983, BSSA, Rost and Thomas, 2002, RG)

First ideas as early as 1920‘s in exploration geophysics! combining clusters of geophones for SNR improvement

In seismological context the development and use of array techniques is closely related to the start of nuclear test ban negotiations in Geneva 1958

Concept: a high number of small arrays to monitor nuclear underground test activities around the world (planned 170 small aperture arrays with 10 sensors)

Seismic arrays: historical context and developments

First experimental arrays from 1960 to 1963 in U.S. and U.K. (VELA program)

But: small array concept could not be realized due to political reasons (array installations blocked)

Therefore: second best solution for detection and verification purposes of nuclear explosions:

Very large arrays at few spots
→ LASA (1965), NORSAR (1971)

Seismic arrays: historical context and developments

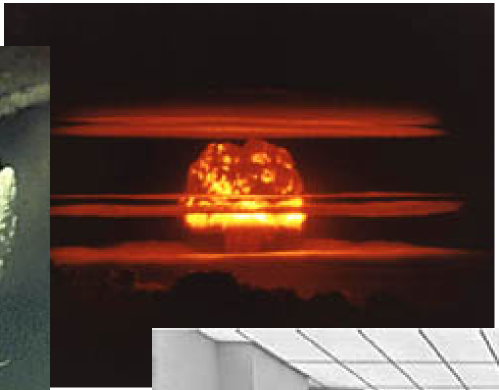
LASA (1965) & NORSAR (1971) facts:

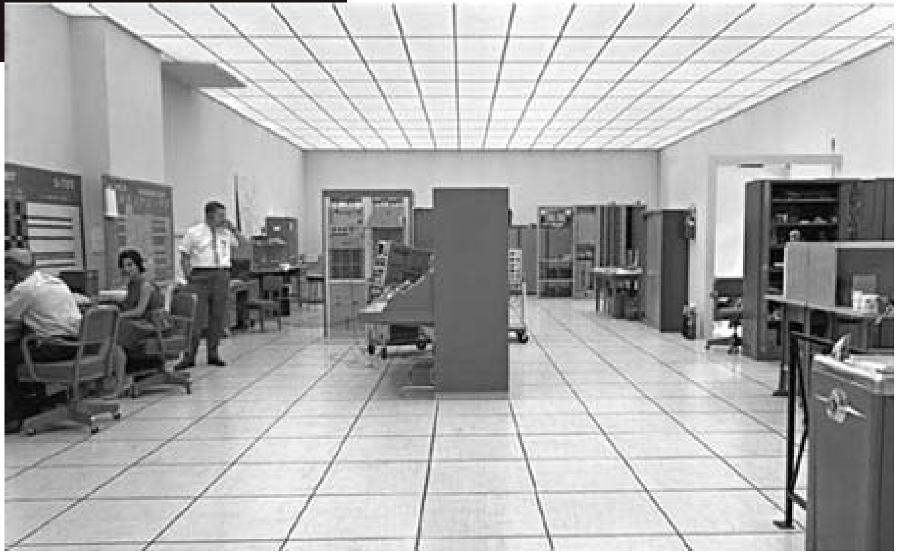
LASA: 200 km aperture, initially 525 stations
NORSAR: 100 km aperture, 198 stations

Huge number of stations →
SNR-improvement for detection/discrimination formidable!
Event location on global scale even for small magnitudes

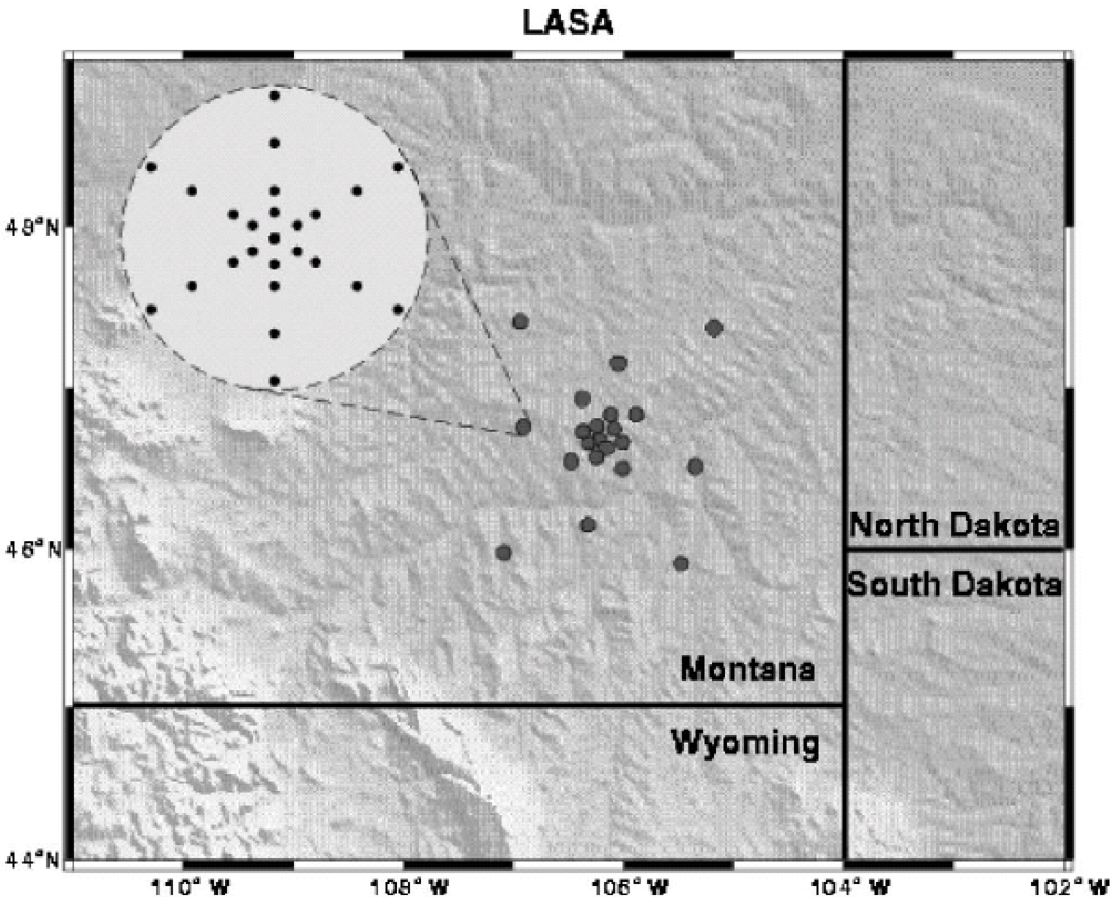
COST OF OPERATION AND MAINTENANCE!!!!!

Seismic arrays: historical context and developments

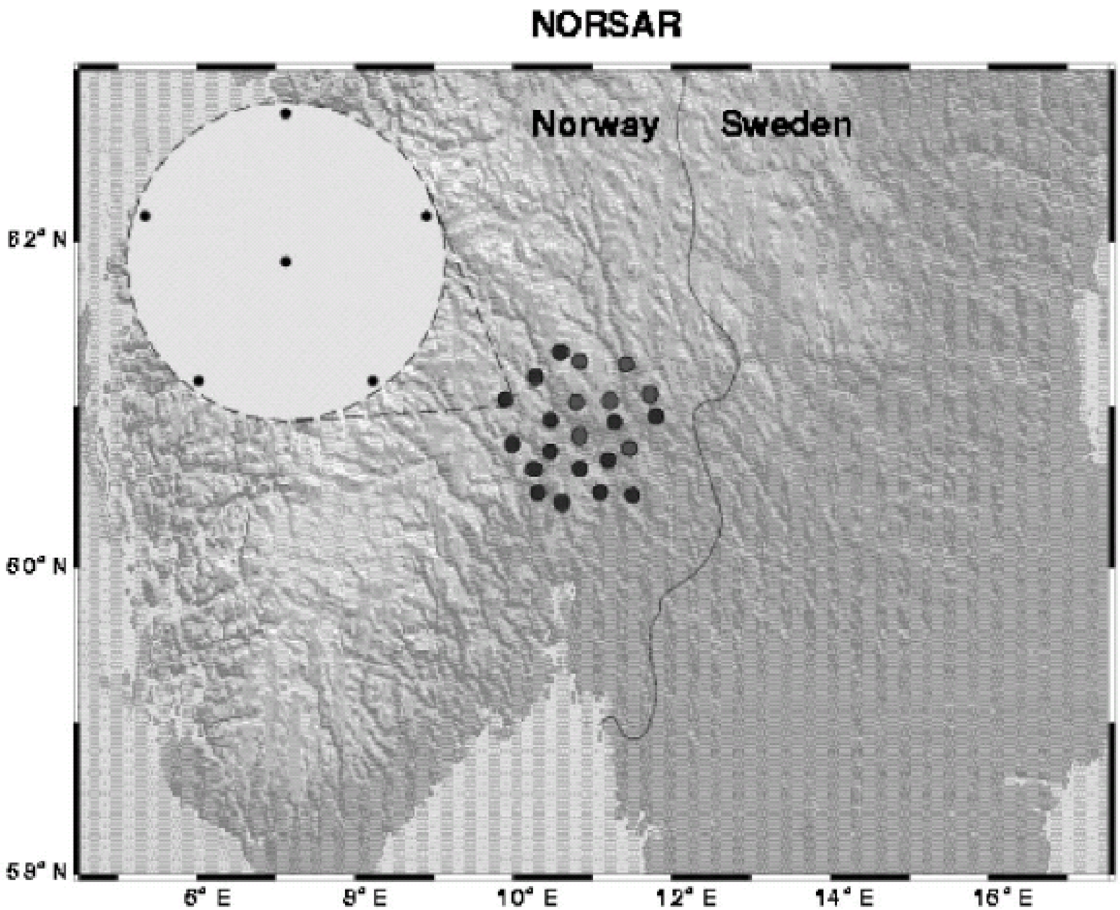




Seismic arrays: historical context and developments



Seismic arrays: historical context and developments



Seismic arrays: historical context and developments

Further application domains:

- structural investigations (global/regional/local)
- seismic exploration
- since relative early times a matter of interest
  → ‚seismic noise'!

(i.e. in the context of array design for monitoring arrays it has been recognized that noise is almost never incoherent and white, but rather colored and shows spatial coherence)

What is noise…?

“ … In order to record seismic signals **it is desirable to know the spectrum of seismic noise** since a priori knowledge of the expected signal-to-noise ratio as a function of frequency can best determine the frequency response characteristics of instrumentation. Also, **since arrays are constructed for the purpose of enhancing the signal-to-noise ratio** it is desirable to know beforehand the **nature of the noise**. Is it random or propagating ? How coherent is it ? **Unfortunately these questions tend to require at least a skeleton array to answer them!** . ...“

Davies, 1973

NORES noise correlation analysis ⇨ coherence lengths

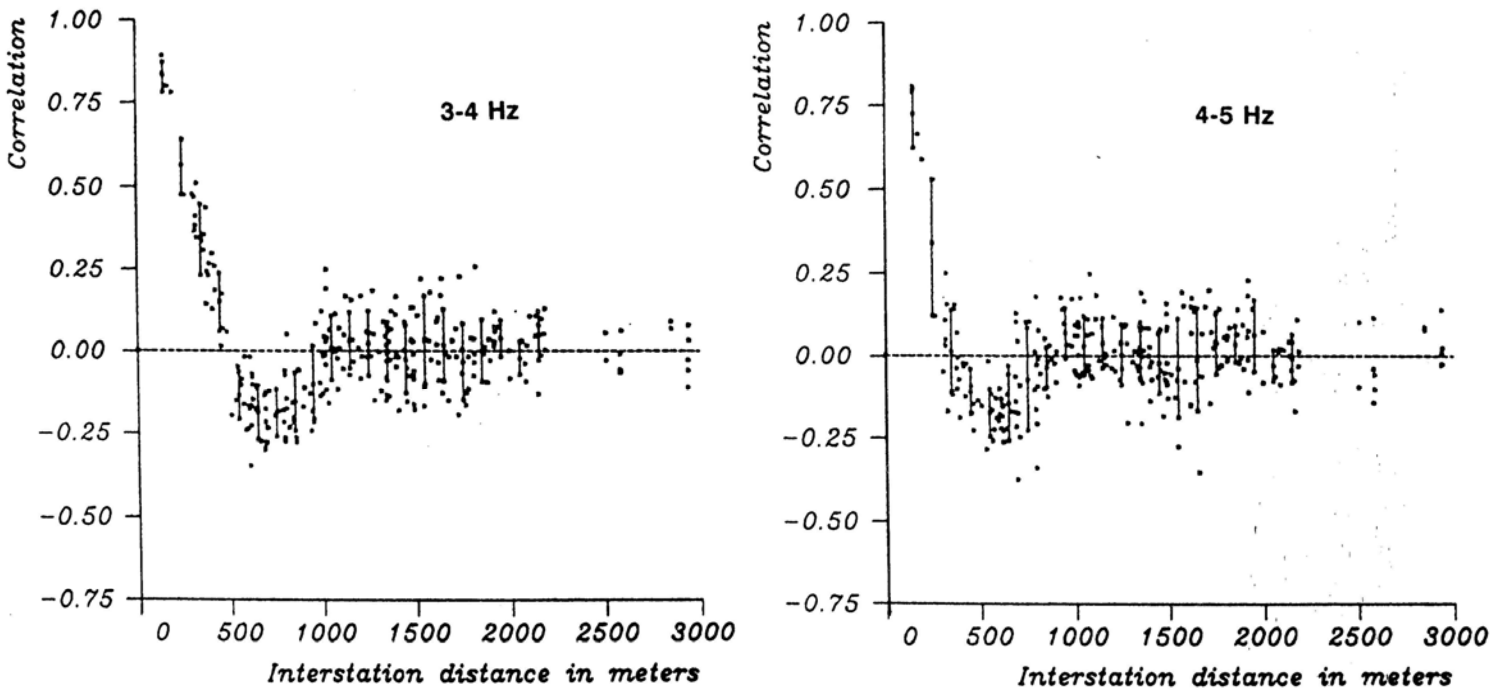
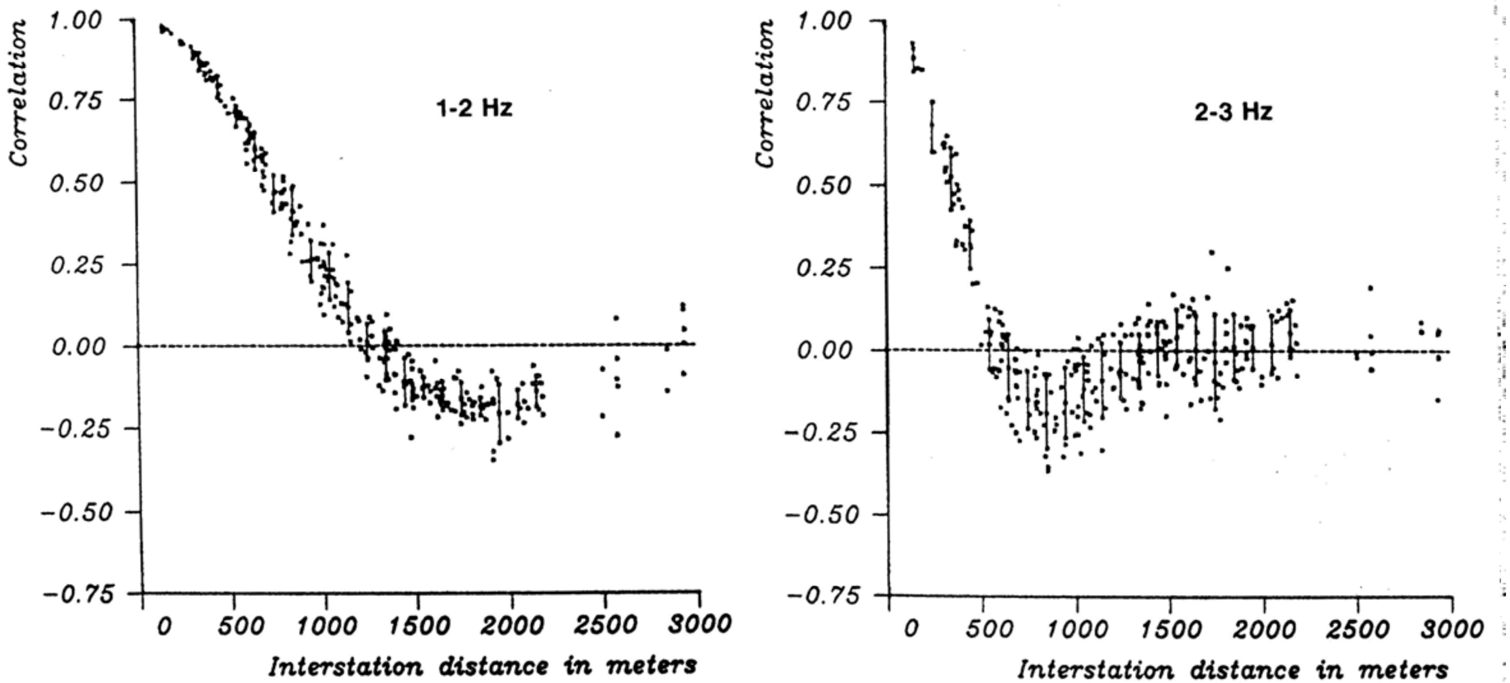


Fig. 2. NORESS noise correlations versus interstation separation for the four frequency bands indicated. The noise segment used is 30 s long and taken at 05.15 h GMT on day 323 of 1985. Mean values and standard deviations within 100 m distance intervals are plotted on top of the population, except for short and long distances, where the number of correlation values is low.

Mykkeltveit, S., K. Åstebøl, D.J. Dornboos & E.S. Husebye (1983): Seismic array configuration optimization. Bull. Seism. Soc. Am., 73: 173-186.

NORES noise correlation analysis ⇨ coherence lengths



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Seismic arrays: historical context and developments

Further application domains:

- since relative early times a matter of interest: ‚seismic noise' → ambient vibrations

K. Aki, 1957, 1965, Toksöz, 1964, Capon et al., 1967
Capon, 1969, Lacoss et al., 1969,
Haubrich and Camy, 1969, Woods and Lintz, 1973,
Henstridge, 1979, Asten and Henstridge, 1984,
Horike, 1985, Tokimatsu et al., 1992, Tokimatsu, 1997

Basic assumption for array processing: the need for a wave propagation model

Simple model and therefore appealing:

Harmonic plane wave representation!

$$D(x,t) = A\exp(i\omega(t \pm x/c)) \qquad D(\vec{x},t) = A\exp(i(\omega t \pm \vec{k}\vec{x}))$$

Particular solution to the homogeneous wave equation

$$\frac{\partial^2 D}{\partial x^2} = \frac{1}{c^2}\frac{\partial^2 D}{\partial t^2}$$

Harmonic plane wave representation

$$D(\vec{x},t) = A\exp(i(\omega t \pm \vec{k}\vec{x}))$$

$$D(\vec{x},t) = A\exp(i\omega(t \pm \vec{u}\vec{x}))$$

phase

positions of constant phase at some time t are wavefronts →
$\vec{k}\vec{x} = const.$ **→ wavefronts are planes in space**
orientation of plane is given by wavenumber vector (normal vector)

Parameters describing plane waves:

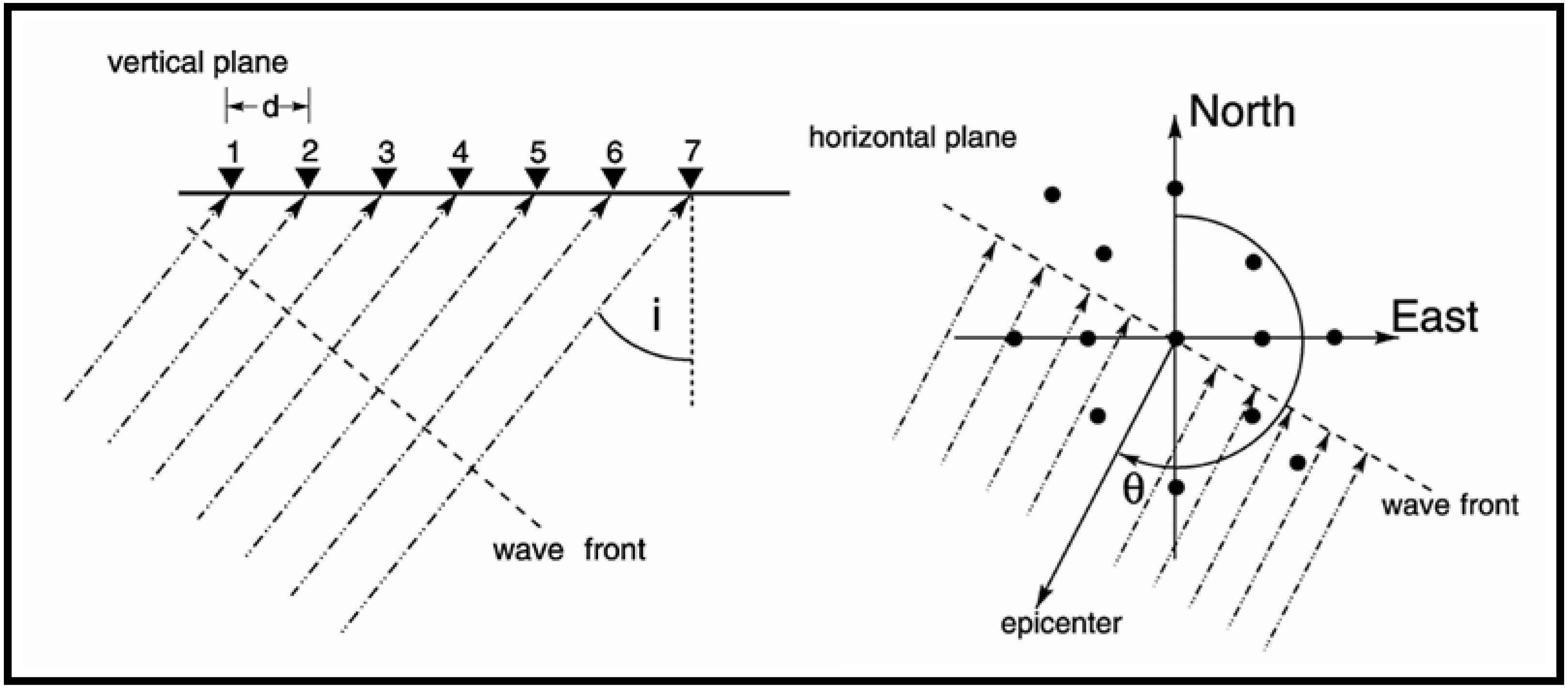
wavenumber $\vec{k} = \omega\vec{u}$ **slowness**

$$k = \frac{2\pi}{\lambda} = \frac{2\pi f}{v} = \frac{\omega}{v} = \omega|\vec{u}|$$

Period + frequency $T = \frac{1}{f} = \frac{2\pi}{\omega}$

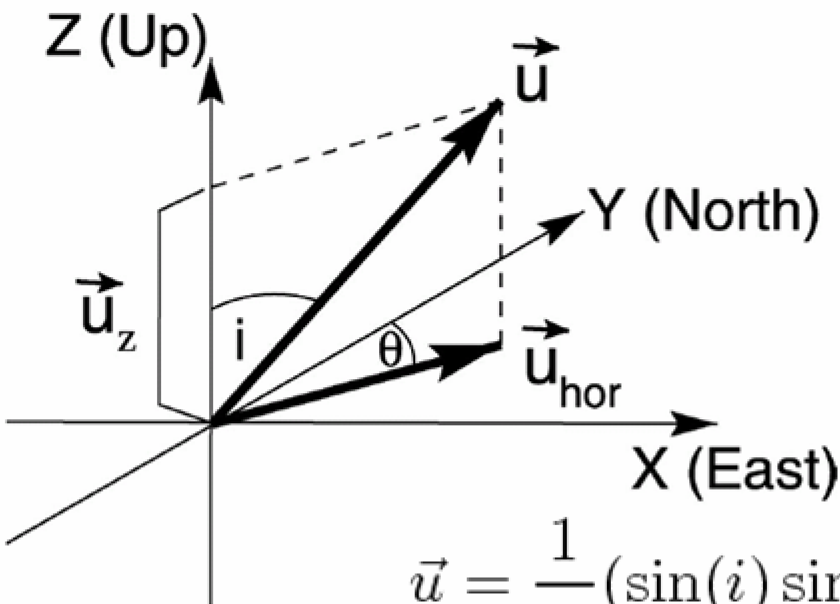
$$v = \lambda f = \lambda\omega/2\pi$$

Basic assumption for array processing: the need for a wave propagation model



Geometry of plane waves – parameters of wave propagation

Basic assumption for array processing: the need for a wave propagation model



slowness vector:

- **in direction of wave propagation**
- **perpendicular to wavefront**
- **length ~ v_0^{-1}**

$$\vec{u} = (u_x, u_y, u_z)$$

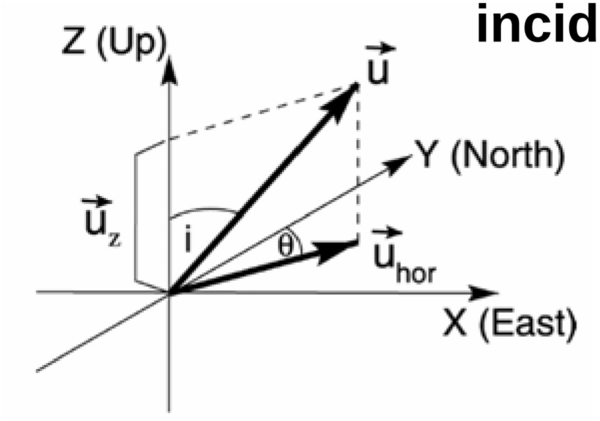
$$\vec{u} = \frac{1}{v_0}(\sin(i)\sin(\theta), \sin(i)\cos(\theta), \cos(i))$$

Geometry of plane waves – parameters of wave propagation

Basic assumption for array processing: the need for a wave propagation model

$$\vec{u} = \frac{1}{v_0}(\sin(i)\sin(\theta), \sin(i)\cos(\theta), \cos(i))$$

slowness vector described by incidence angle i, propagation azimuth θ

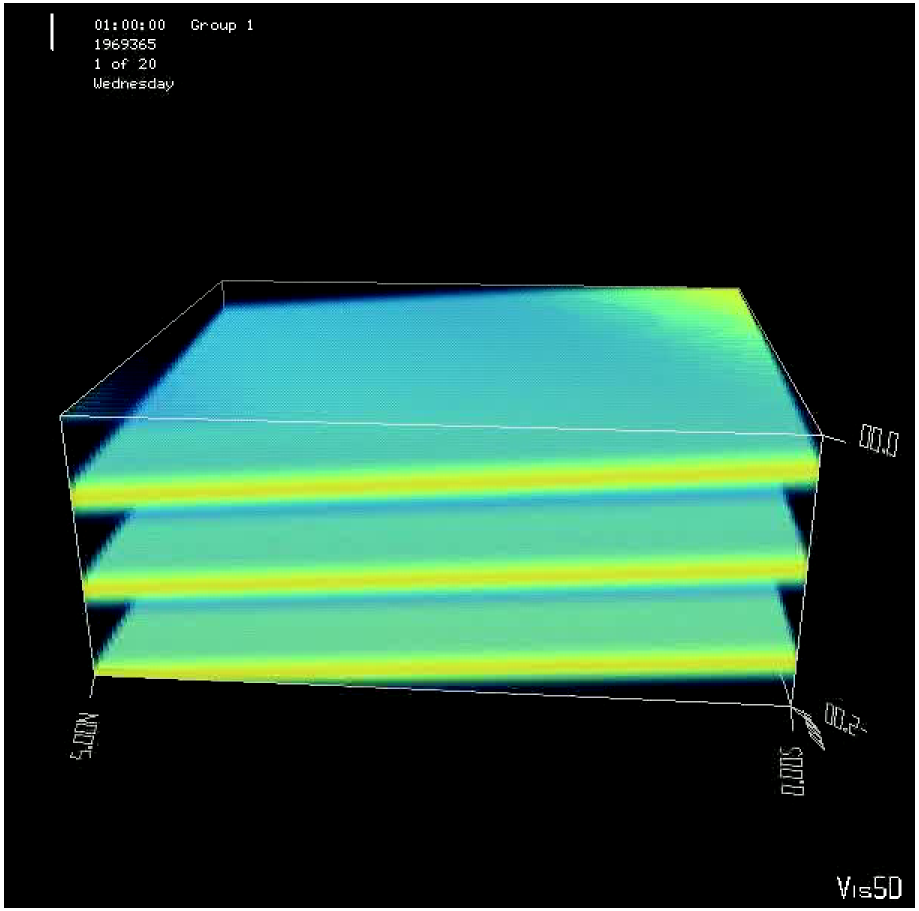


$$\vec{u} = u_{hor}(\sin(\theta), \cos(\theta), \frac{1}{\tan(i)})$$

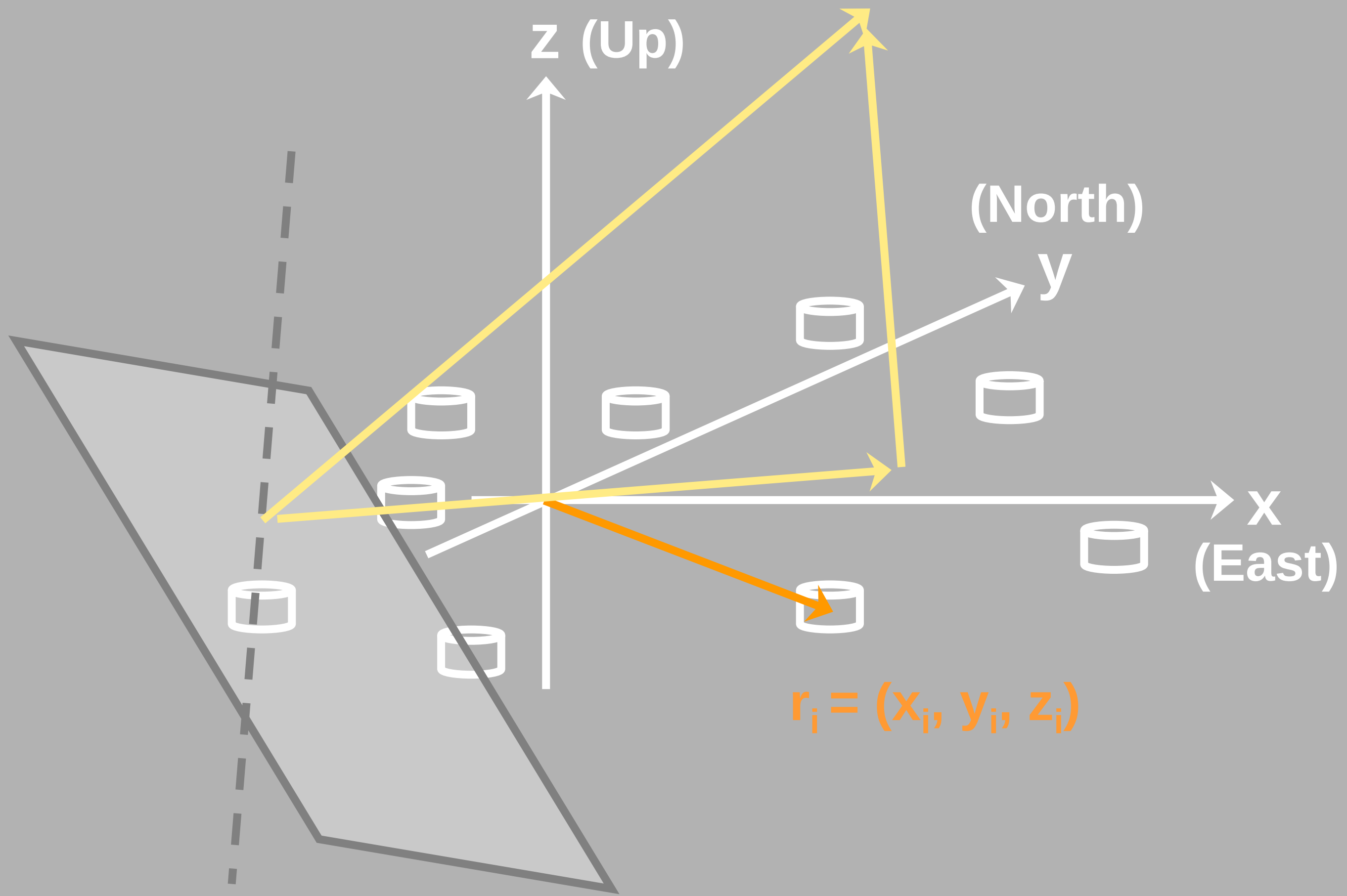
$$|\vec{u}_{hor}| = \frac{1}{v_{app}} = \frac{\sin(i)}{v_0} = p$$

Geometry of plane waves – parameters of wave propagation

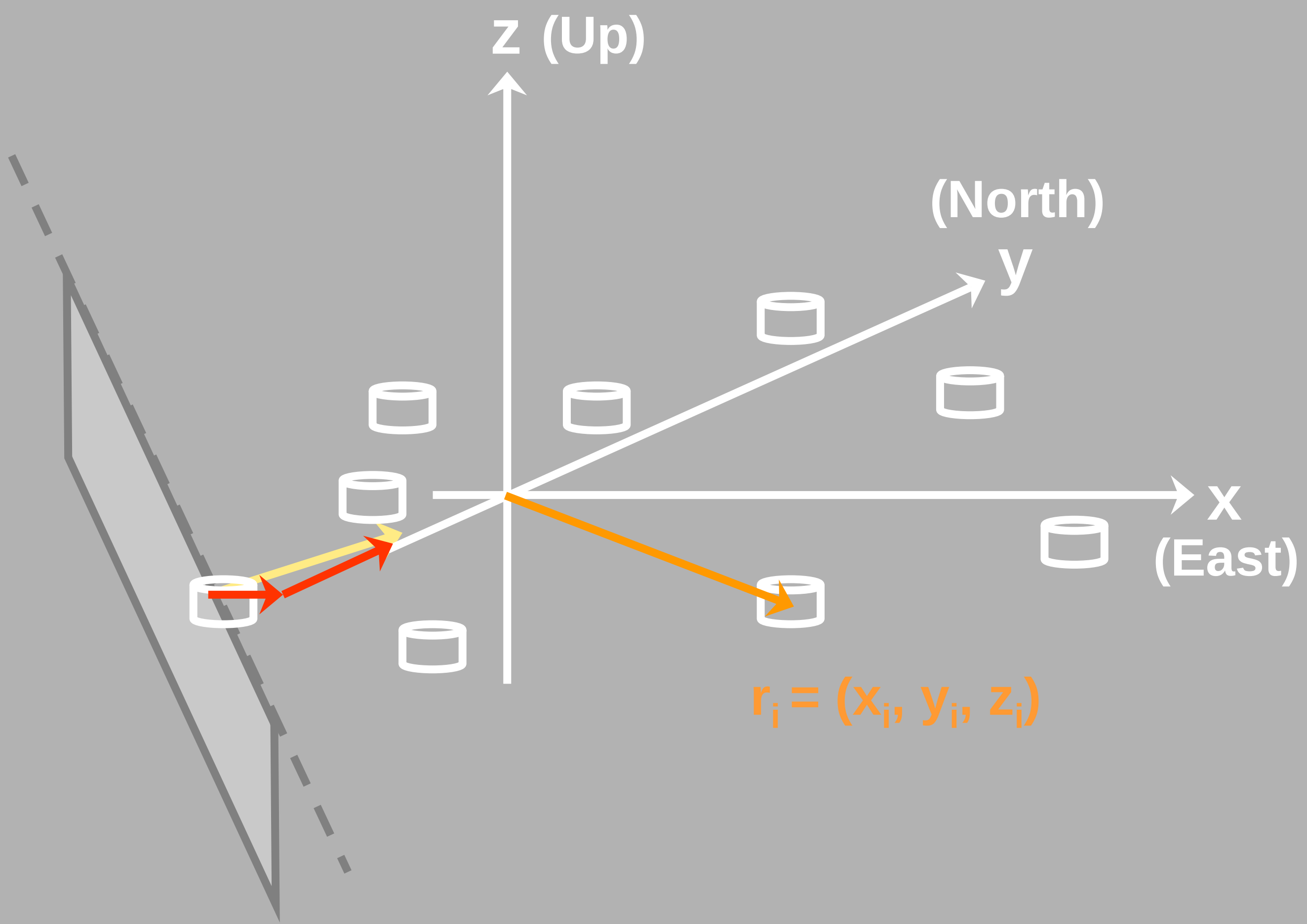
Basic assumption for array processing: the need for a wave propagation model



Plane wave propagation model: body wave type

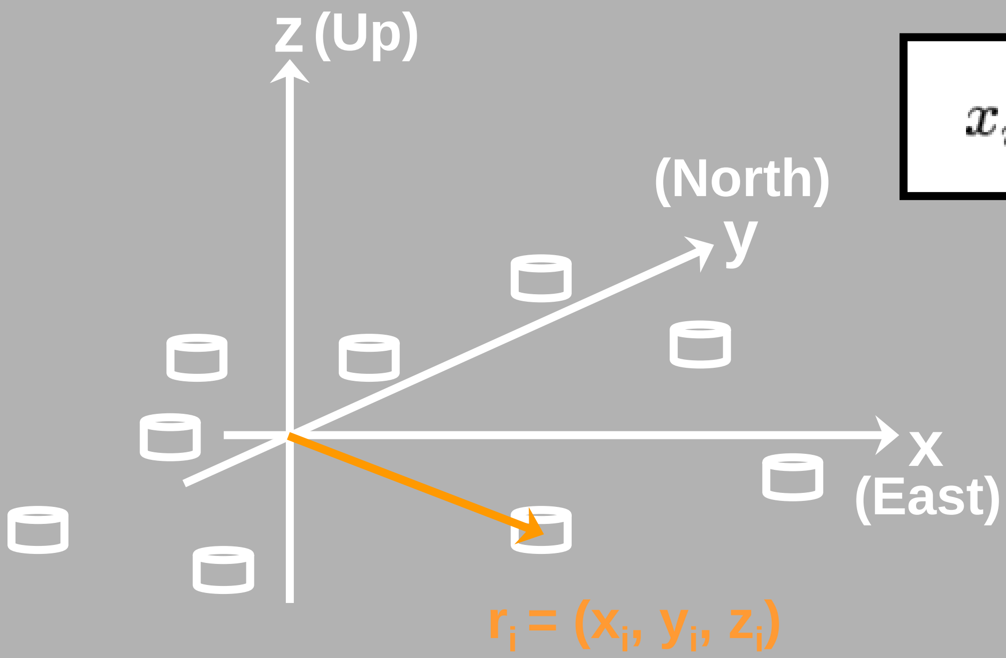


Plane wave propagation model: surface wave type



Plane wave propagation model: observation of particular waveform s(t) at array sensors

Plane wave description at a single sensor:

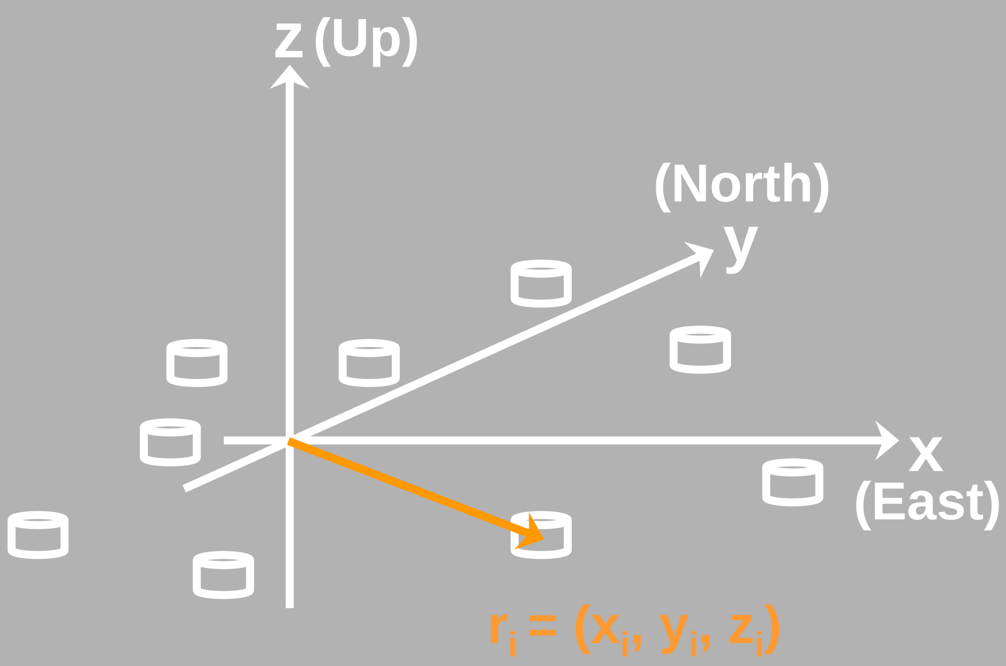


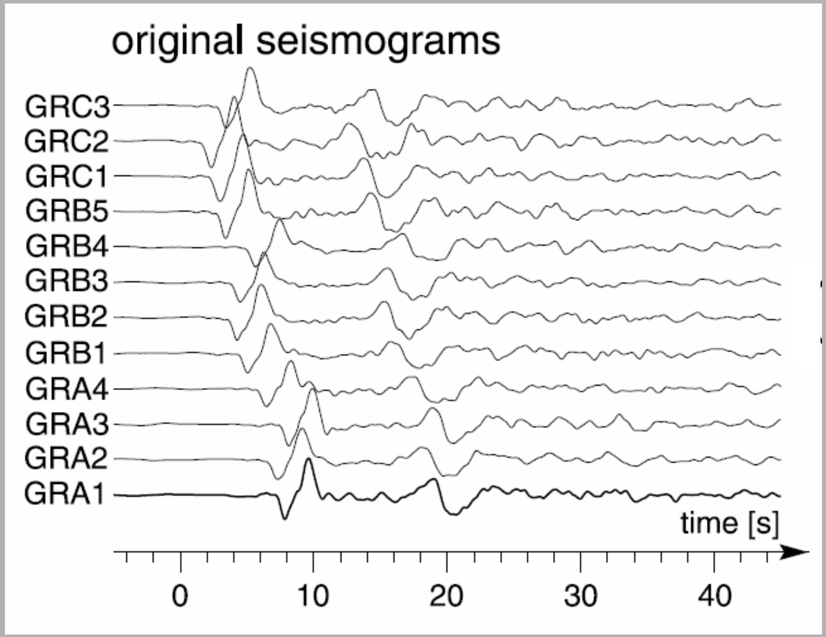
$$\boxed{x_i(t) = s(t - \vec{r}_i \vec{u}_{hor}) + n_i(t)}$$

relative time shifts depend on station position AND horizontal slowness

Plane wave propagation model: array recordings

Example plane wave seismogram section:

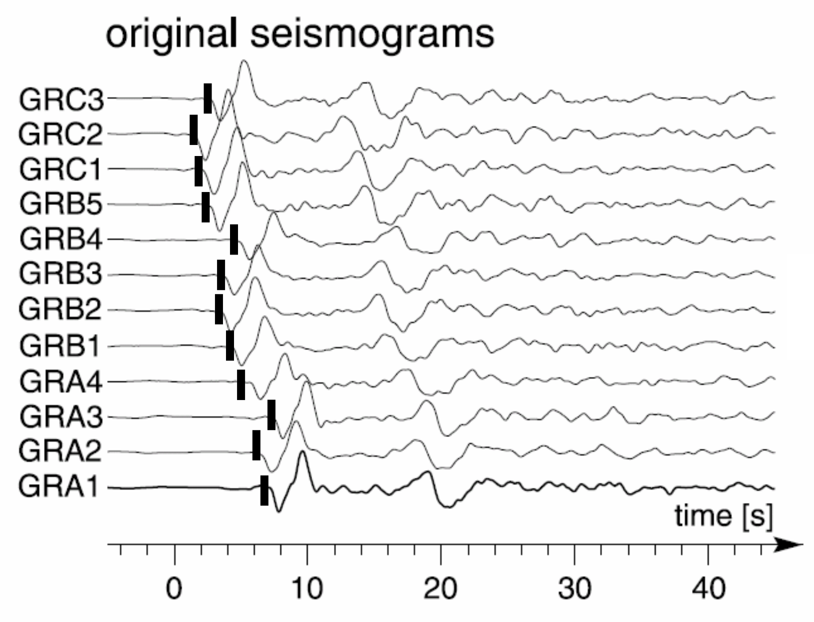




$$x_i(t) = s(t - \vec{r}_i \vec{u}_{hor}) + n_i(t)$$

Plane wave parameter determination I: transient signals with high SNR

Arrival time picking at N stations



Standard procedure

Plane wave parameter determination I: transient signals with high SNR

Arrival time at station i $$t_i = t_o + \vec{u}_{hor}\vec{r}$$

+ plane wave model → set of linear equations

$$\begin{bmatrix} t_1 - t_o \\ t_2 - t_o \\ \vdots \\ t_N - t_o \end{bmatrix} = \begin{bmatrix} r_{1x} & r_{1y} \\ r_{2x} & r_{2y} \\ \vdots & \vdots \\ r_{Nx} & r_{Ny} \end{bmatrix} \begin{bmatrix} u_x \\ u_y \end{bmatrix}$$

Plane wave parameter determination I: transient signals with high SNR

In short: $\vec{t} = \underline{R}\vec{u}_{hor}$

formal solution: $\vec{u}_{hor} = \underline{R}^{-1}\vec{t}$

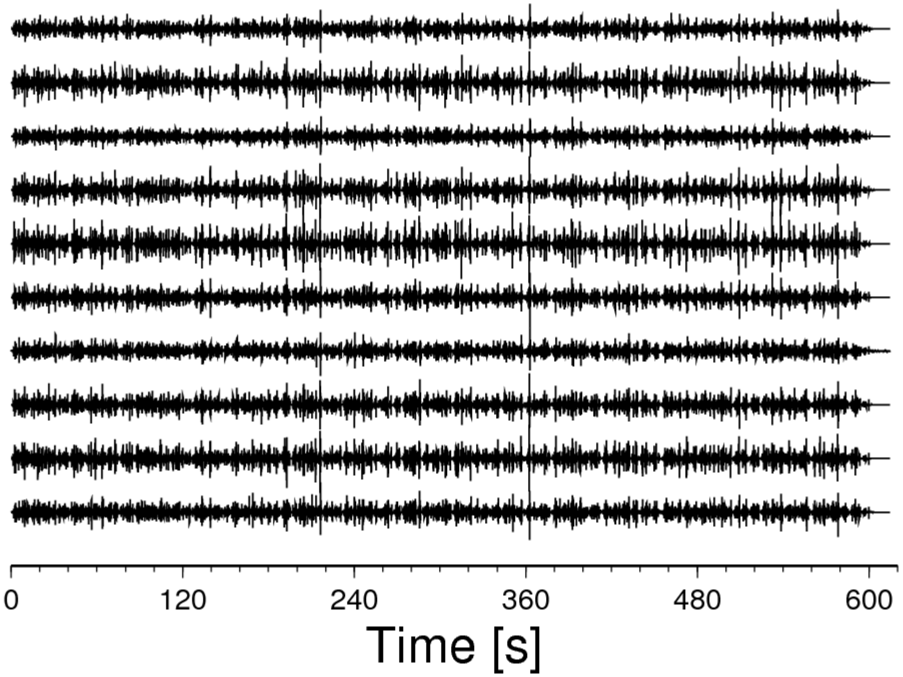
we get (e.g. by LSQ)

$$p = |\vec{u}_{hor}| \qquad \theta = atan(u_x/u_y)$$

Inverse of app. velocity / propagation azimuth

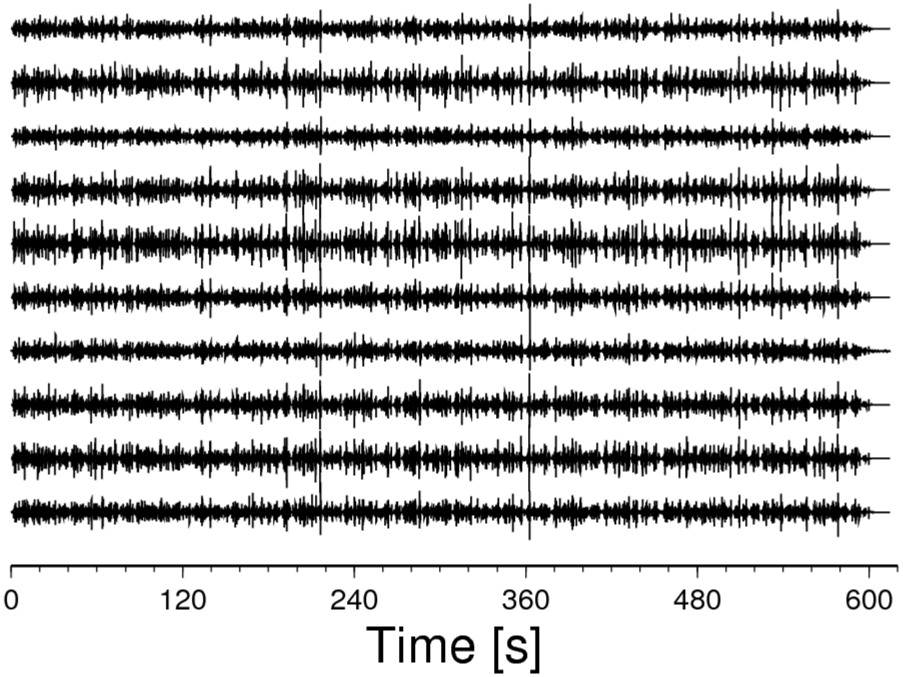
Plane wave parameter determination II: enhancing signals for specific parameters

Question: is there a signal with parameters θ_0, p_0 ?



Plane wave parameter determination II: enhancing signals for specific parameters

Answer: let‘s try!



Plane wave parameter determination II: enhancing signals for specific parameters

Answer: let‘s try!

$$\vec{u} = u_{hor}(\sin(\theta), \cos(\theta), \frac{1}{\tan(i)})$$

observation

$$x_i(t) = s(t - \vec{r}_i \vec{u}_{hor}) + n_i(t)$$

delay observation

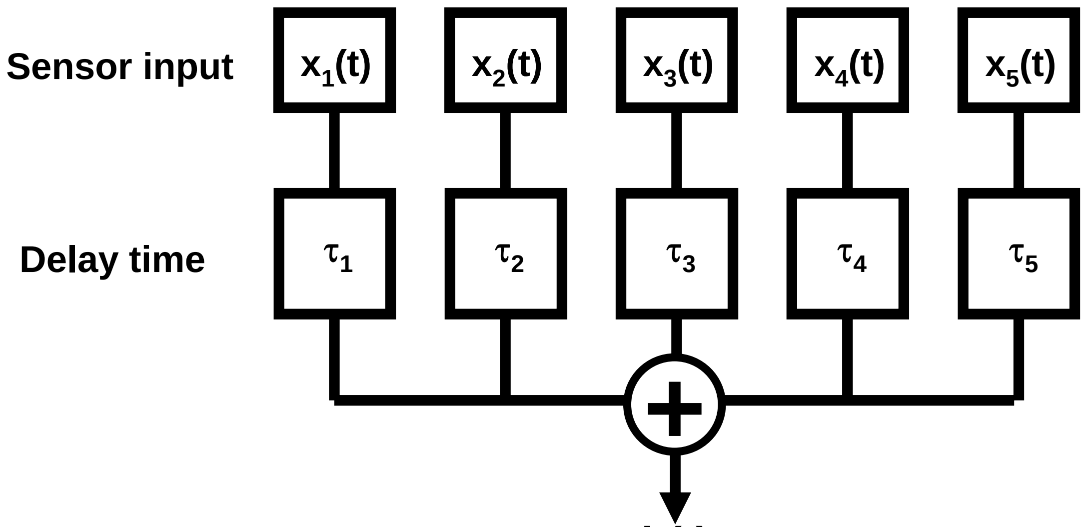
$$\tilde{x}_i(t) = x_i(t + \vec{r}_i \vec{u}_{hor})$$

$$\tilde{x}_i(t) = s(t) + n_i(t + \vec{r}_i \vec{u}_{hor})$$

and sum $$b(t) = \frac{1}{N}\sum_{i=1}^{N} \tilde{x}_i(t) = s(t) + \frac{1}{N}\sum_{i=1}^{N} n_i(t + \vec{r}_i \vec{u}_{hor})$$

uncorrelated noise is suppressed by $\sqrt{N}$ (at best)

Plane wave parameter determination II: enhancing signals for specific parameters

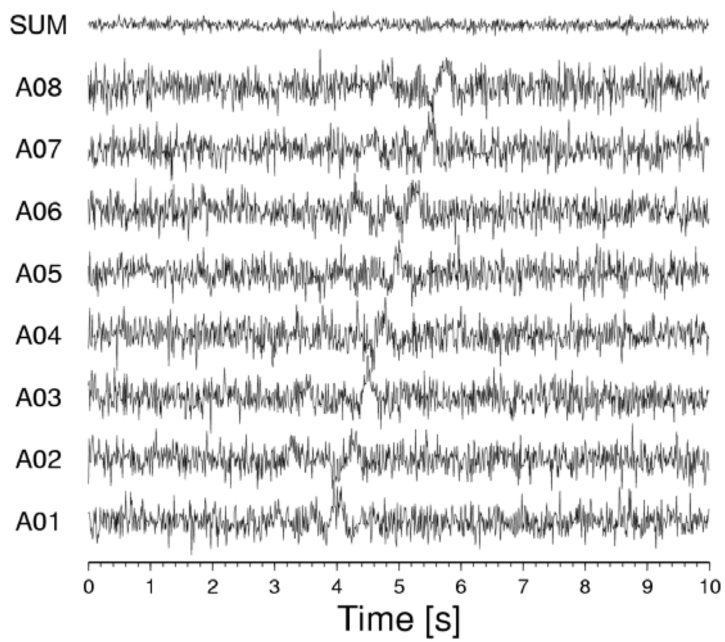


Delay-and-sum Beam = b(t) = Array output

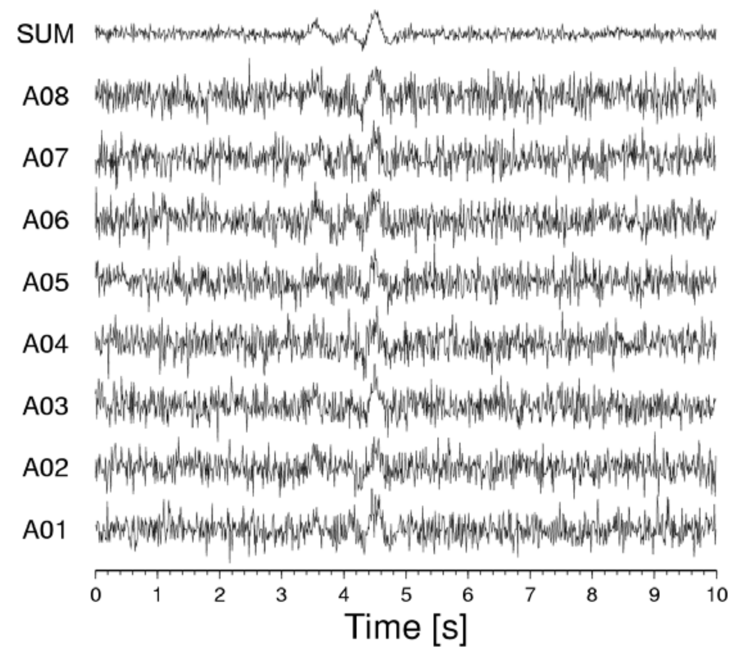
$$b(t) = \frac{1}{N}\sum_{i=1}^{N} \tilde{x}_i(t) = s(t) + \frac{1}{N}\sum_{i=1}^{N} n_i(t + \vec{r}_i \vec{u}_{hor})$$

Plane wave parameter determination II: enhancing signals for specific parameters

with some p, θ



with p_0, θ_0



Plane wave parameter determination II: enhancing signals for specific parameters

Delay and sum beamformer – how well did it work?

Quantify by power measure... → beam energy

$$E(beam) = \sum_{k=1}^{N} \left| b(k\Delta t) \right|^2$$

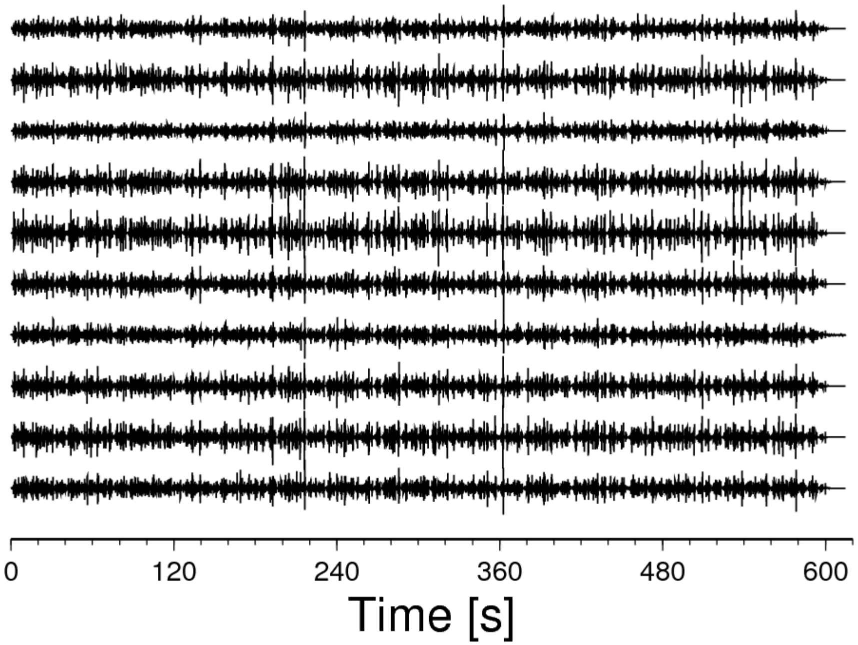
Quantify by coherence measure... → semblance

$$S = \frac{\sum_{j=-M/2}^{j=M/2} \left| \sum_{i=1}^{N} \tilde{x}_i(t_j) \right|^2}{N \sum_{j=-M/2}^{j=M/2} \sum_{i=1}^{N} \left| \tilde{x}_i(t_j) \right|^2}$$

semblance = filter output / filter input energy ratio

Plane wave parameter determination III: any signal arriving with any possible parameter

Question: is there some signal with arbitrary parameter θ, p which we might be interested in (e.g. Rayleigh waves...)?



Plane wave parameter determination III: any signal arriving with any possible parameter

Answer: let's do the same procedure as before – now we just have to search for different values of θ, p

GRIDSEARCH technique!

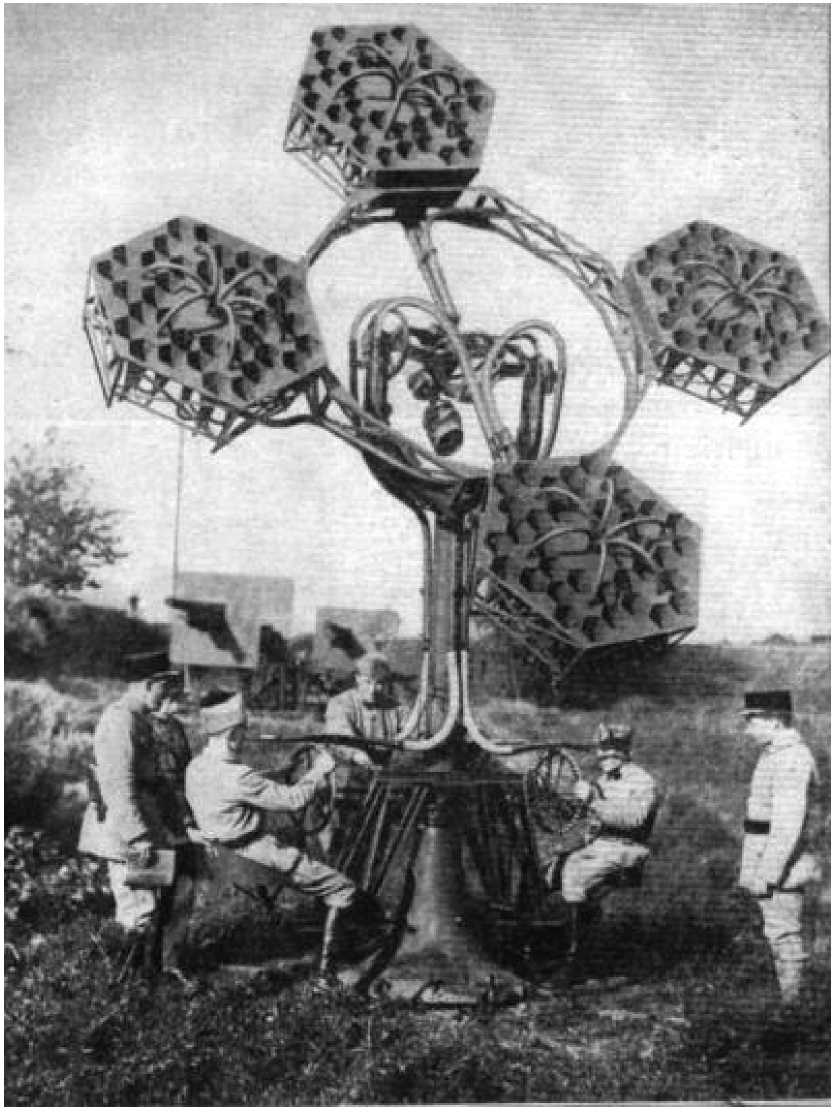
As quantitative measure of goodness of fit, we can use: Beampower (or Semblance)

Delay and sum beamforming → Slowness power spectrum
Slowness power spectrum: Beampower as function of p and θ
Other standard tool in array analysis:
Vespagram: Beampower as function of p for constant θ

A humanoid delay-and-sum beamformer


Professor Mayer's topophone [1880]

Professor Mayer's topophone 1880



Plane wave parameter determination IV: Beamforming ... a different view

Noisefree signal at station i: $$x_i(t) = s(t - \vec{r}_i \vec{u}_0)$$

Signal propagates with true (horizontal) slowness vector $\vec{u}_0$

Beamforming according to test slowness vector $\vec{u}$
→ time shifted traces:

$$\tilde{x}_i(t) = x_i(t + \vec{r}_i(\vec{u} - \vec{u}_0))$$

Beam: $$b(t) = \frac{1}{N} \sum_{i=1}^{N} x_i(t + \vec{r}_i(\vec{u} - \vec{u}_0))$$

Parseval theorem:

$$E(beam) = \int_{-\infty}^{\infty} b^2(t)dt = \int_{-\infty}^{\infty} |B(\omega)|^2 d\omega$$

Plane wave parameter determination IV: Beamforming ... a different view

Time domain ←→ frequency domain: Fourier transform

Shifting theorem of FT: $x(t - t_0) \Leftrightarrow X(f)\exp(2\pi j f t_0)$

Then we get for the beam energy in frequency domain:

$$E(beam) = \int_{-\infty}^{\infty} |B(\omega)|^2 \, d\omega$$

$$E(beam) = \int_{-\infty}^{\infty} \left| \frac{1}{N} \sum_{i=1}^{N} \tilde{X}_i(\omega) \right|^2 d\omega$$

$$E(beam) = \int_{-\infty}^{\infty} \left| \frac{1}{N} \sum_{i=1}^{N} X_i(\omega) \exp(j\omega \vec{r}_i(\vec{u} - \vec{u}_0)) \right|^2 d\omega$$

Plane wave parameter determination IV: Beamforming ... a different view

$$E(beam) = \int_{-\infty}^{\infty} |X_i(\omega)|^2 \left| \frac{1}{N} \sum_{i=1}^{N} \exp(j\omega \vec{r}_i(\vec{u} - \vec{u}_0)) \right|^2 d\omega$$

$$E(\vec{u} - \vec{u}_0) = \int_{-\infty}^{\infty} |X_i(\omega)|^2 \, |A(\vec{u} - \vec{u}_0, \omega)|^2 \, d\omega$$

Array response function:

$$A(\vec{u} - \vec{u}_0, \omega) = \left| \frac{1}{N} \sum_{i=1}^{N} \exp(j\omega \vec{r}_i(\vec{u} - \vec{u}_0)) \right|$$

Plane wave parameter determination IV: Beamforming ... a different view → ARRAY RESPONSE

Array response function:

in slowness:

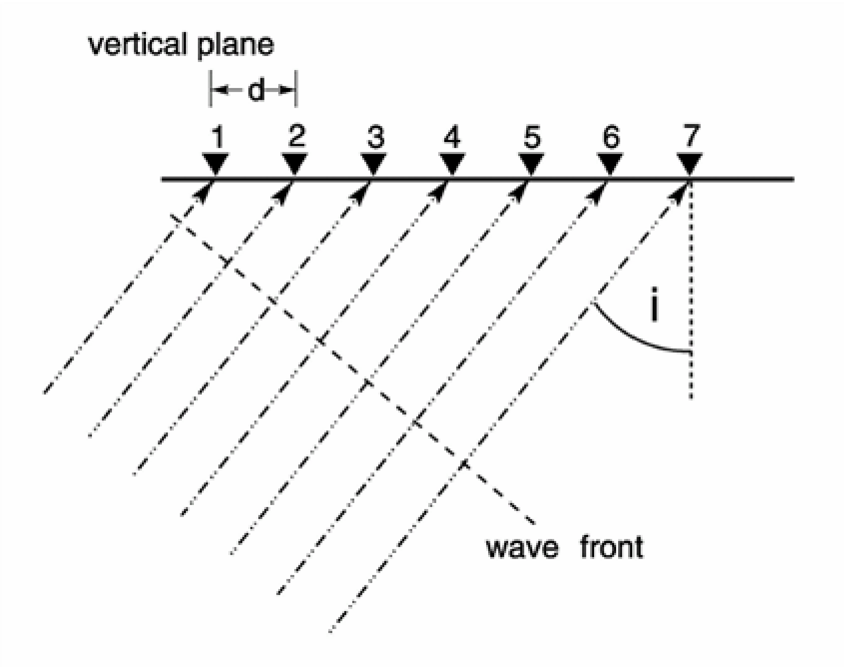
$$A(\vec{u}-\vec{u}_0,\omega)=\left|\frac{1}{N}\sum_{i=1}^{N}\exp(j\omega\vec{r}_i(\vec{u}-\vec{u}_0))\right|$$

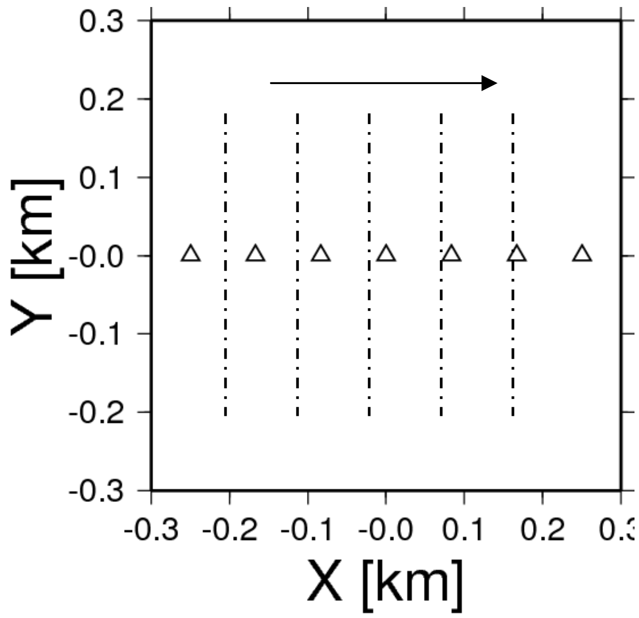
in wavenumber:

$$A(\vec{k}-\vec{k}_0)=\left|\frac{1}{N}\sum_{i=1}^{N}\exp(j\vec{r}_i(\vec{k}-\vec{k}_0))\right|$$

Estimating the quality / capabilities of an array from its array response

Array response function, starting with simplest layout ....





Array response – parametrization of array geometry
Starting simple – 1D line of receivers, spaced equidistantly

For the linear array example, we need only 2 parameters to describe the array geometry:

d_{min} = interstation distance

N = number of sensors

$(N-1)d_{min} = D_{max}$ = Aperture

Station positions are then uniquely defined by $\vec{r}_i \rightarrow i d_{min}$

In this linear problem the wavenumber vector reduces to its x-component: $\vec{k} - \vec{k}_0 \rightarrow k_x - k_0$

and therefore the wavenumber response:

$$\left| A(\vec{k} - \vec{k}_0) \right| = \left| A(k_x - k_0) \right|$$

Array response – parametrization of array geometry
Starting simple – 1D line of receivers, spaced equidistantly

The 1D array response is then written as:

$$|A(k_x - k_0)| = \left| \frac{1}{N} \sum_{i=1}^{N} \exp(jid_{min}(k_x - k_0)) \right|$$

Note: expression is periodic in x-component

Periodicity at:

$$k_{sidelobe} = 2\pi / d_{min} \Rightarrow u_{sidelobe} = 1/(fd_{min})$$

width of main lobe: $2\pi/((N-1)d_{min}) \rightarrow 2\pi/D_{max}$

Array response – 1D layout – width of mainlobe?

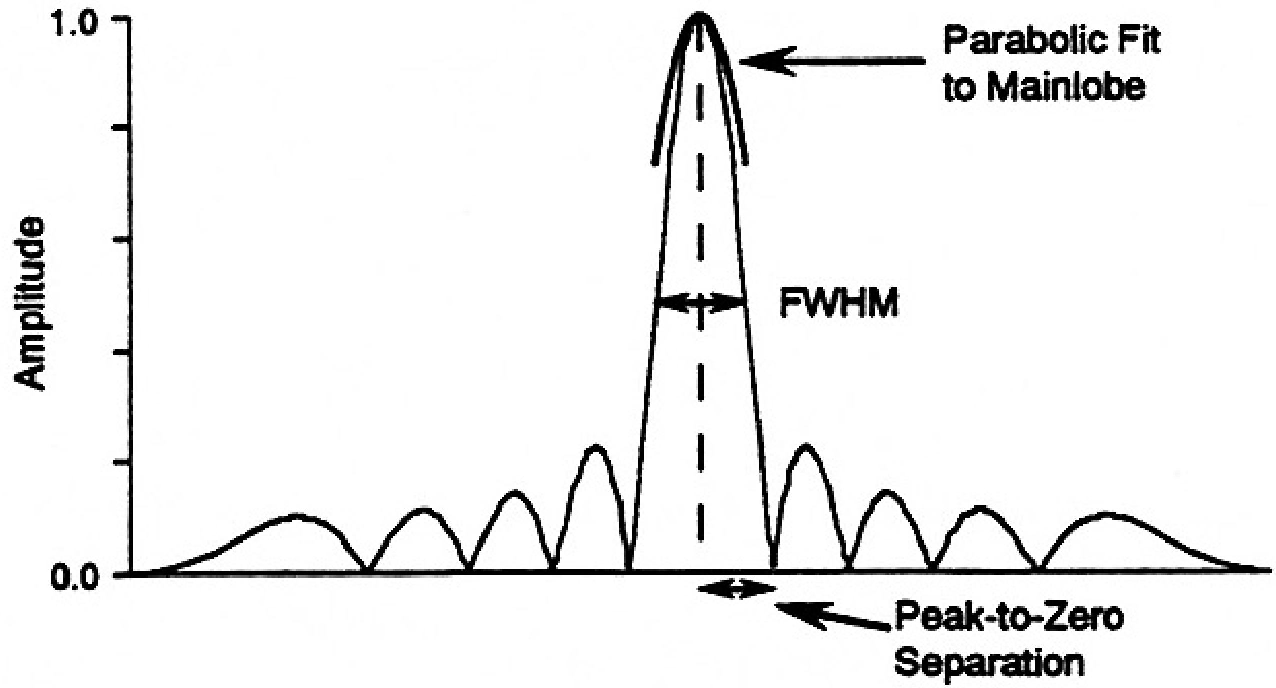
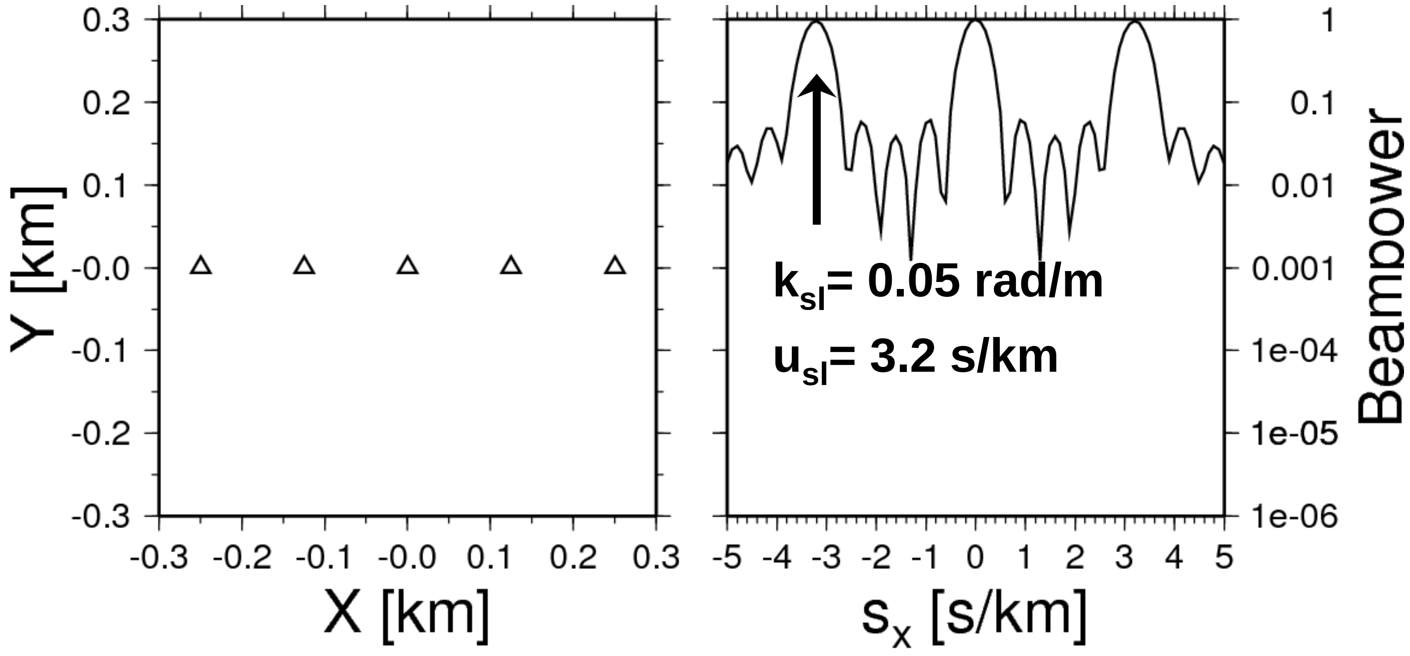


Figure 4.10 A typical array pattern is shown. The wavenumber resolution is defined as the reciprocal of the width of the mainlobe, but "width" may be defined in several ways. For symmetric array patterns, the simplest is the peak to first zero. A second measure is *FWHM* (full-width, half-maximum). Asymmetric array patterns are best characterized by the parabolic width.

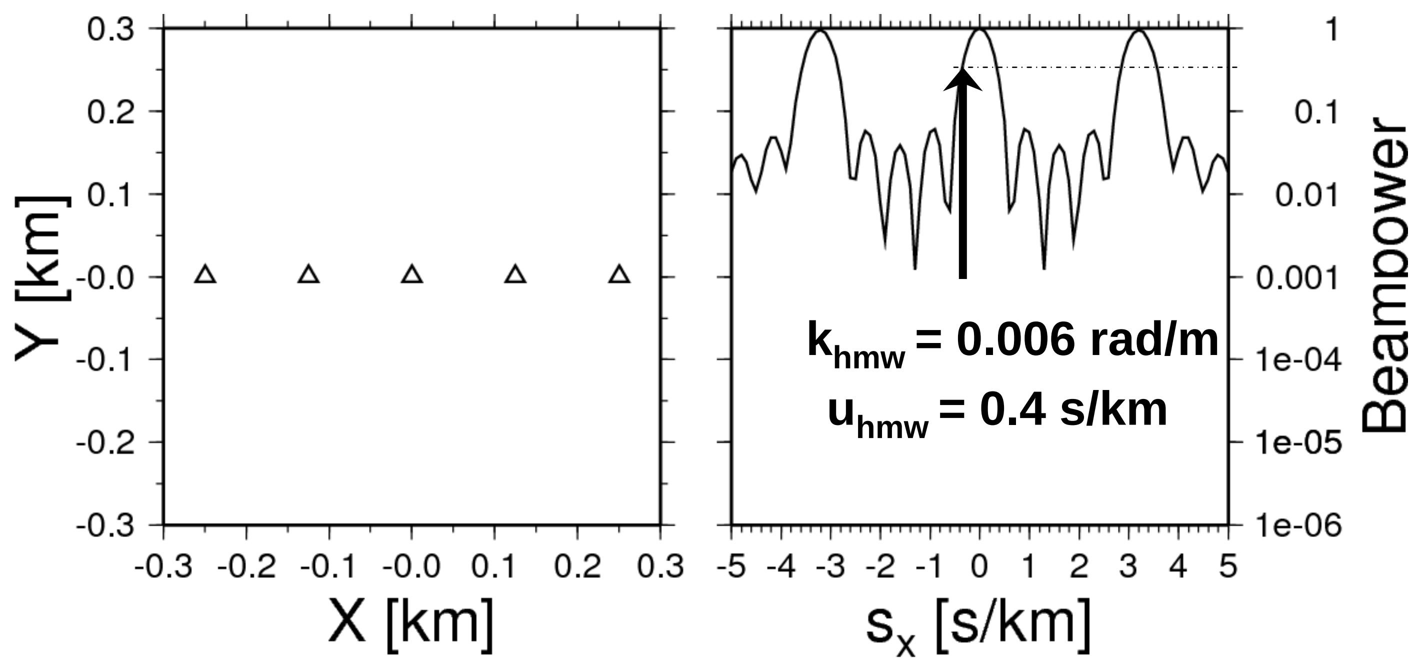
Array response – parametrization of array geometry
Starting simple – 1D line of receivers, spaced equidistantly

Example: N = 5 sensors, d_{min} = 125 m (in slowness @2.5 Hz)



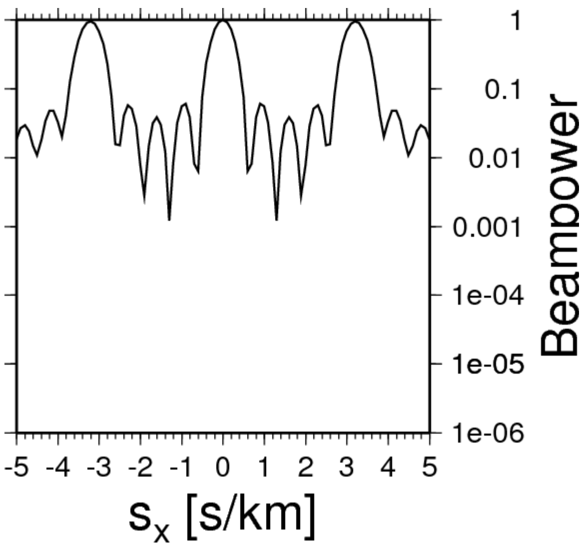
Array response – parametrization of array geometry
Starting simple – 1D line of receivers, spaced equidistantly

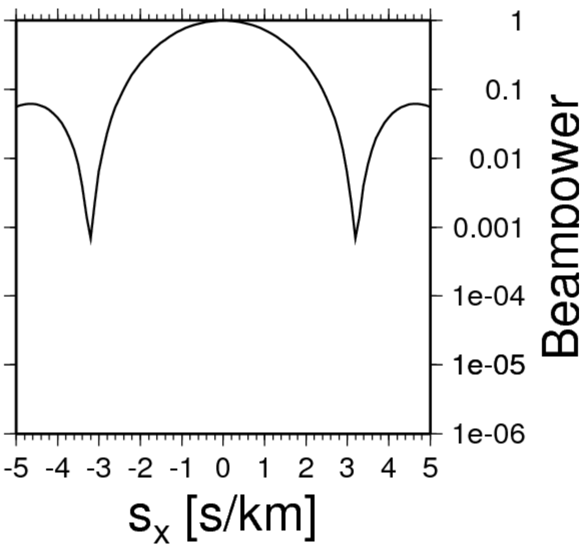
Example: N = 5 sensors, d_{min} = 125 m (in slowness @2.5 Hz)



Array response – parametrization of array geometry
1D layout – parameter influence – interstation distance d_{min}

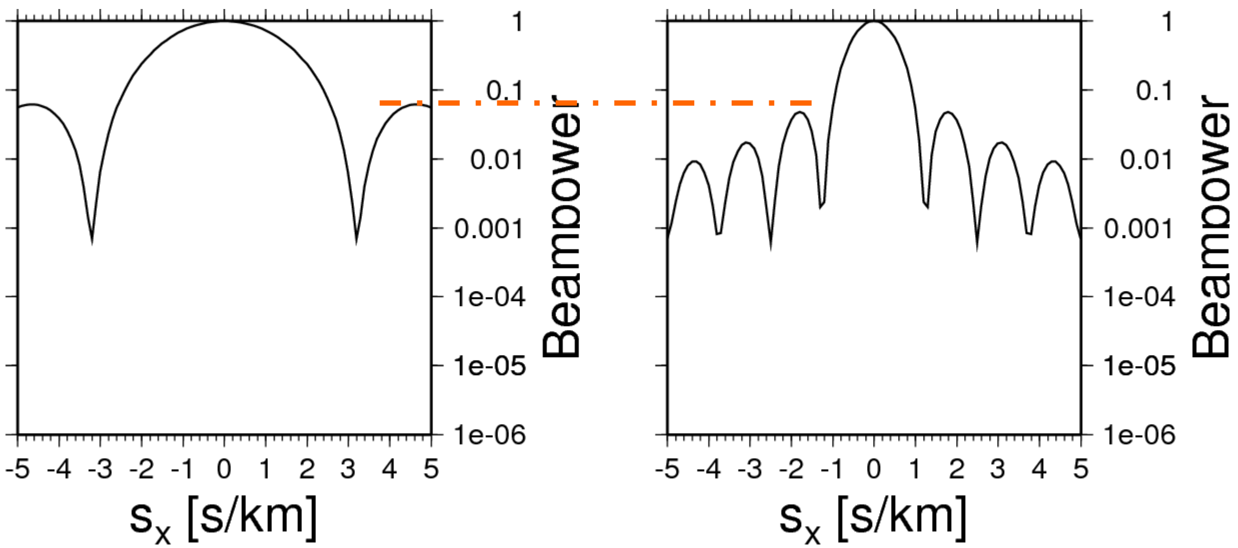
N = 5, d_{min} = 125 m → d_{min} = 25 m



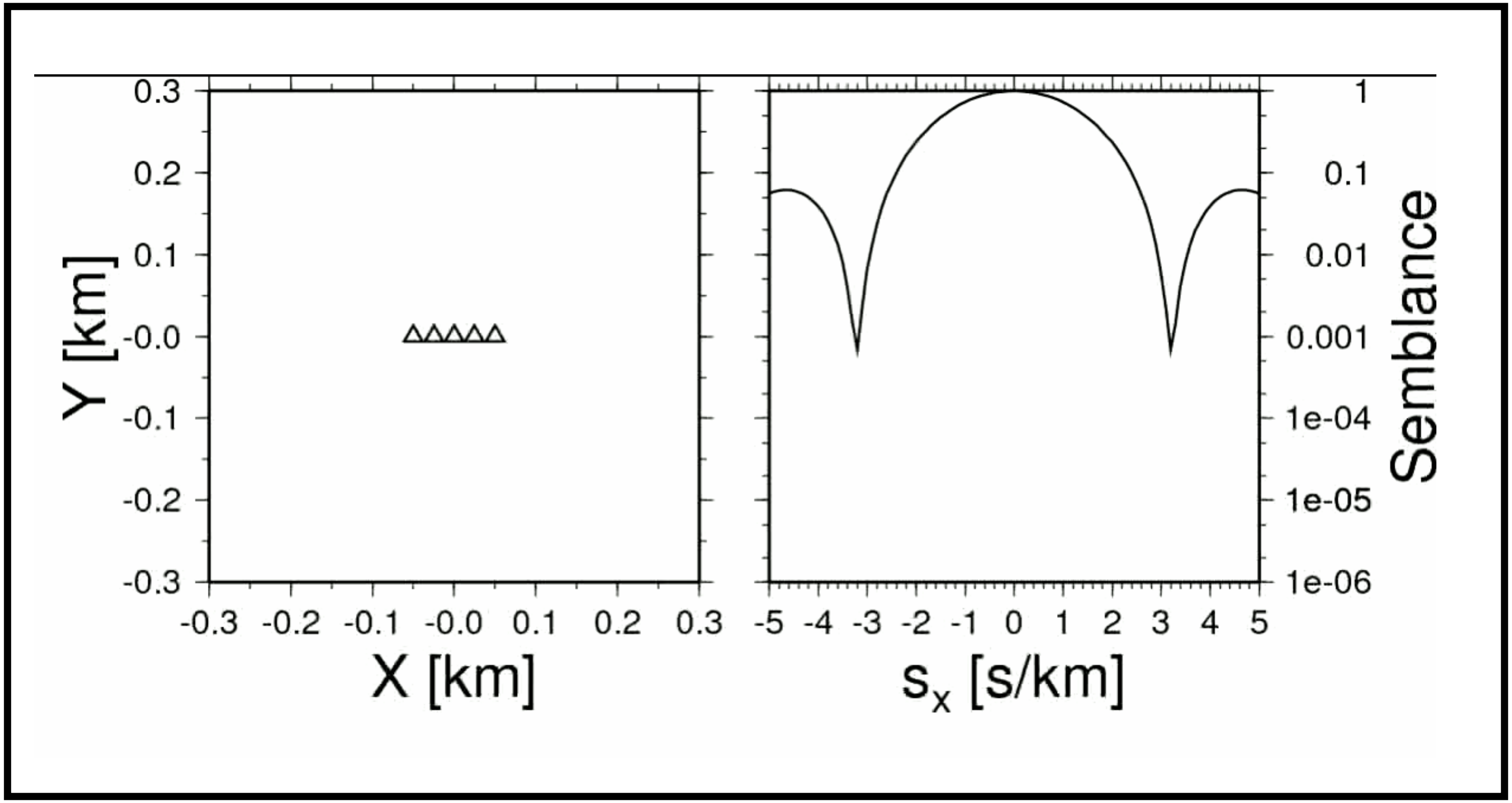


Array response – parametrization of array geometry
1D layout – parameter influence – number of stations N

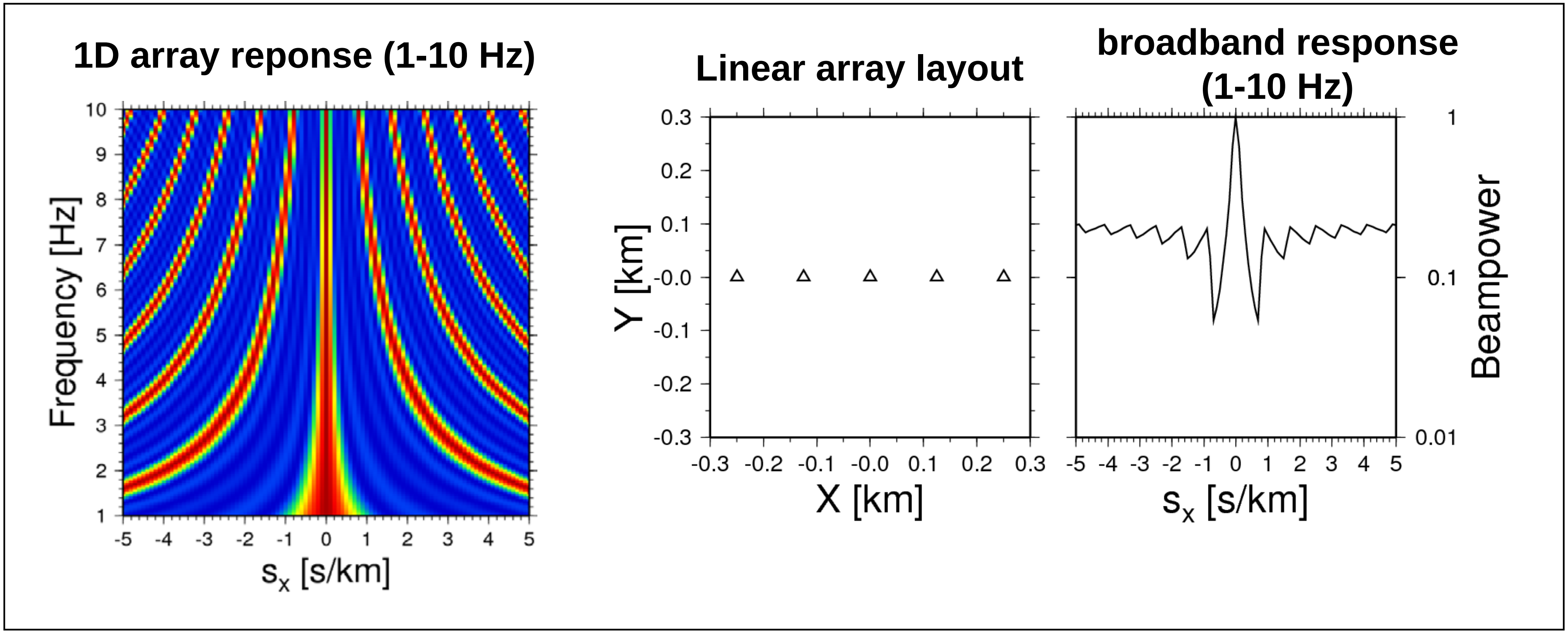
d_{min} = 25 m, N = 5 → N = 15



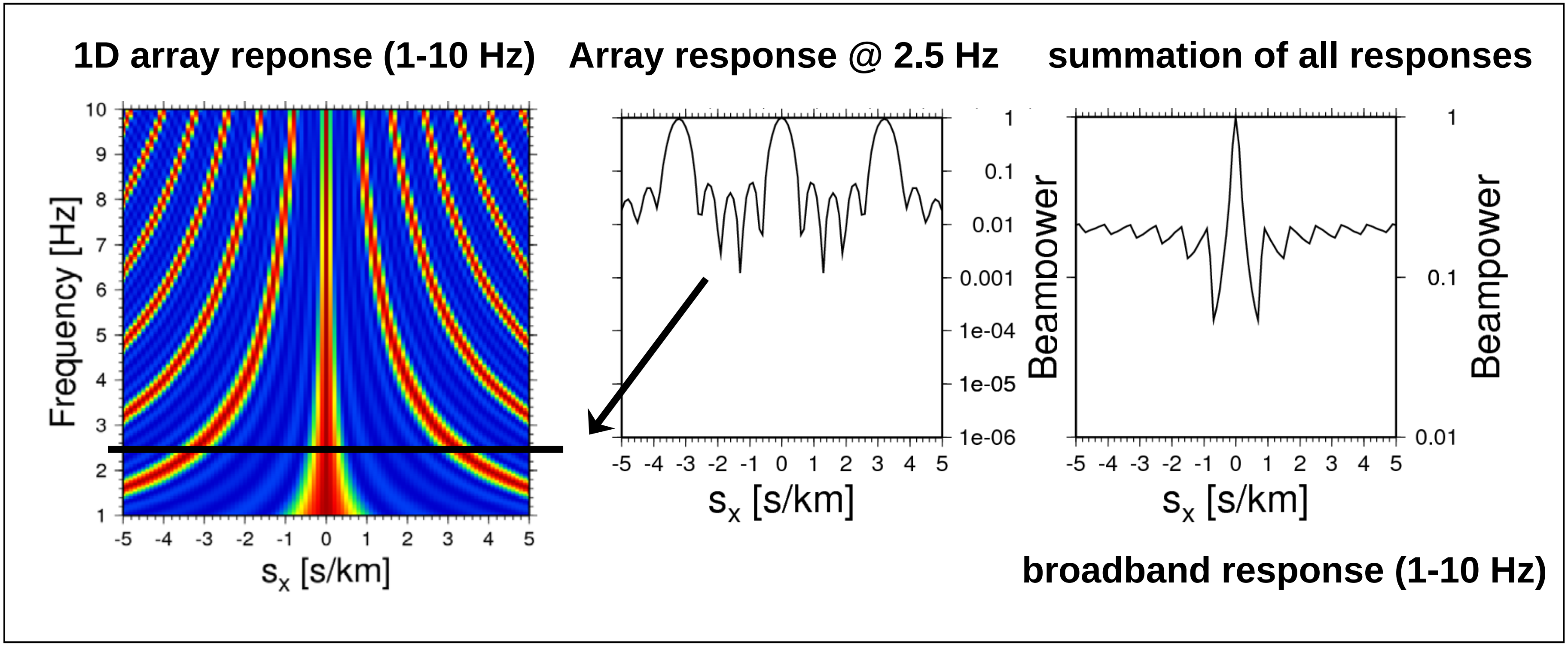
Array response – parametrization of array geometry
1D layout – parameter influence...



Array response – 1D layout – parameter influence broadband frequency wavenumber approach



Array response – 1D layout – parameter influence broadband frequency wavenumber approach



Array response – slowness or wavenumber?

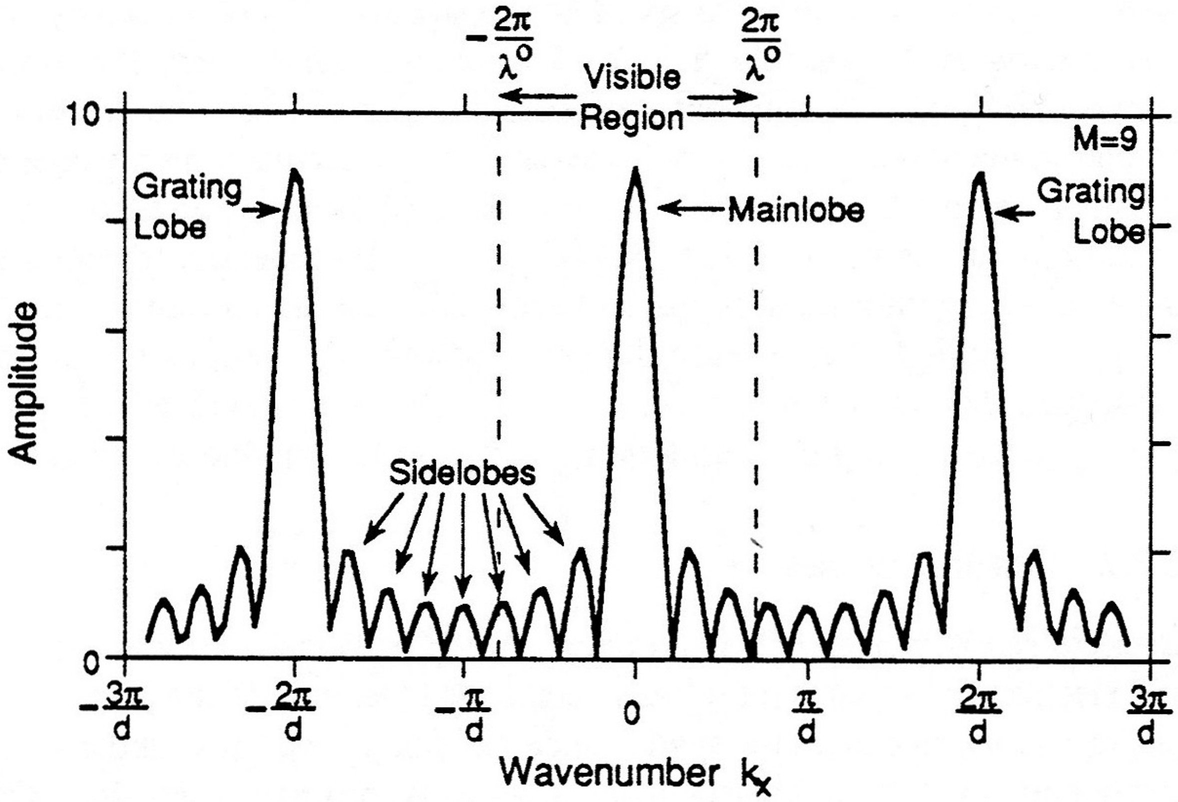


Figure 3.20 The aperture smoothing function magnitude $|W(k)|$ for uniform shading is plotted for a nine-sensor regular linear array. This spatial spectrum has period $k = 2\pi/d$. The visible region of the aperture smoothing function is that part for which $-2\pi/\lambda^o \leq k_x^o \leq 2\pi/\lambda^o$. What might be called secondary mainlobes—those not located at the origin—are termed grating lobes.

Array geometry and discrete spatial sampling of a continuous wavefield

Array measurements can be seen as:

a discrete spatial sampling of a continuous process

sensor locations

seismic wavefield (1D/2D/3D projection)

For 1D linear arrays with equidistant spacing the equivalence to time series sampling is easy to recognize

Array geometry and discrete spatial sampling of a continuous wavefield

discrete spatial sampling of a continuous process

consequences: **aliasing (sampling theorem)**

at least 3 samples per period, wavelength

time domain $\Delta T < T_{min}/2$

spatial domain $\Delta x < \lambda^{\star}_{min}/2$ *** apparent**

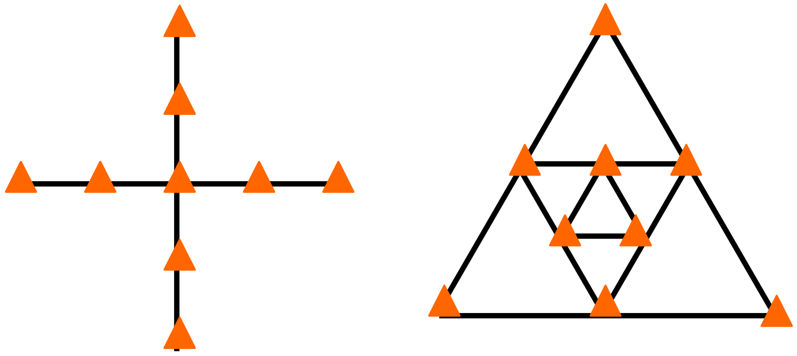
spectral resolution limit

time domain $\Delta\omega = 2\pi/((N-1)\Delta T)$

spatial domain $\Delta k = 2\pi/((N-1)d_{min}) = 2\pi/D_{max}$

Array response
Extension to 2D situation – planar arrays

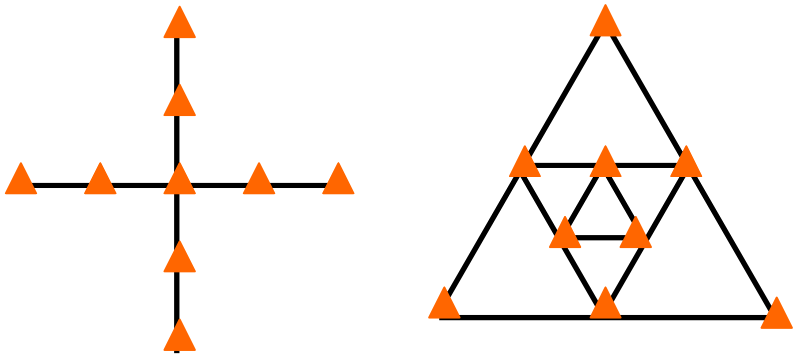
$$A(\vec{u}-\vec{u}_0,\omega)=\left|\frac{1}{N}\sum_{i=1}^{N}\exp(j\omega\vec{r}_i(\vec{u}-\vec{u}_0))\right|$$



$$A(\vec{k}-\vec{k}_0)=\left|\frac{1}{N}\sum_{i=1}^{N}\exp(j\vec{r}_i(\vec{k}-\vec{k}_0))\right|$$

Array response
Extension to 2D situation – planar arrays

similar story as for 1D-layouts,
BUT parametrization more difficult

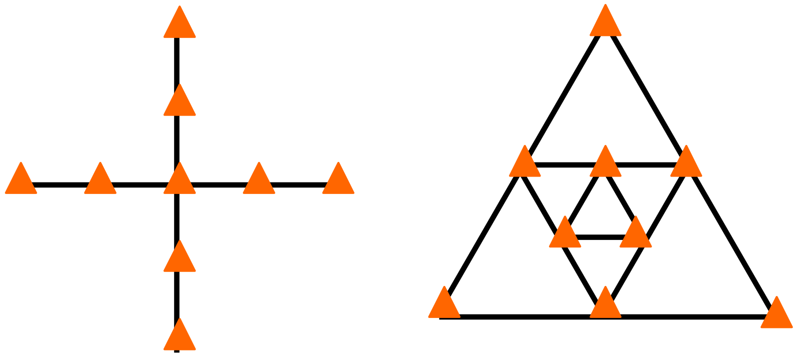


d_{min}, N, D_{max} (aperture)

Array response
Extension to 2D situation – planar arrays

N clear

BUT: d_{min} and D_{max} show directional dependence

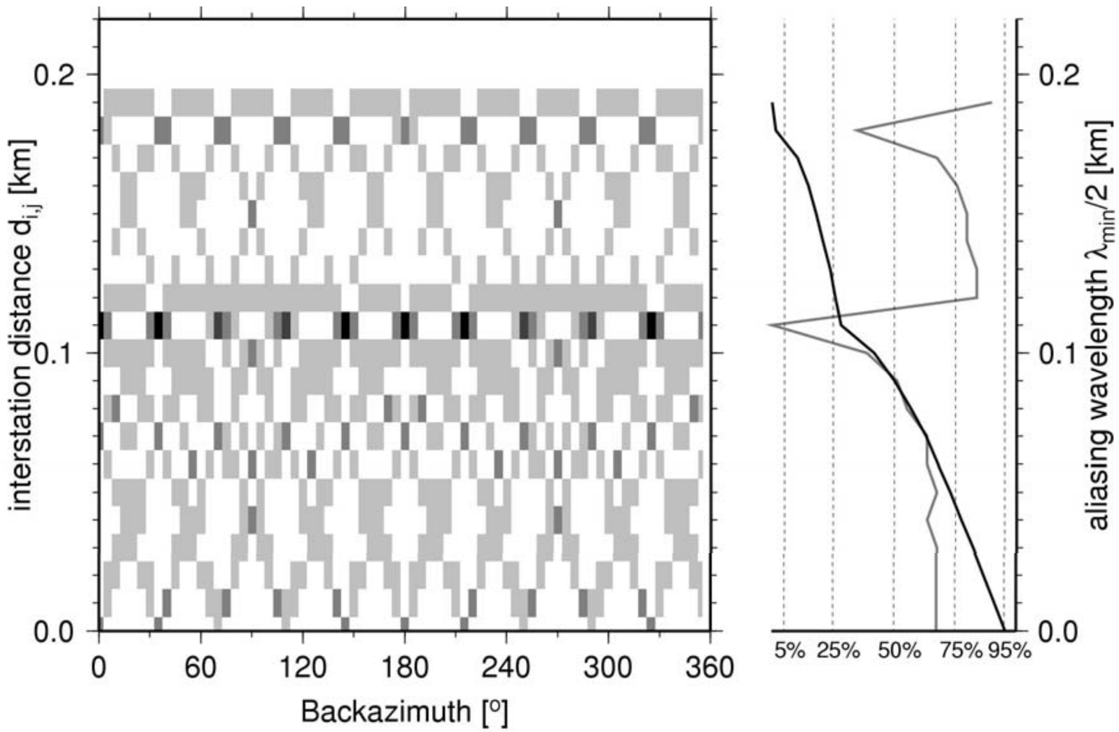


especially there will always be some direction, in which d_{min} is vanishing!

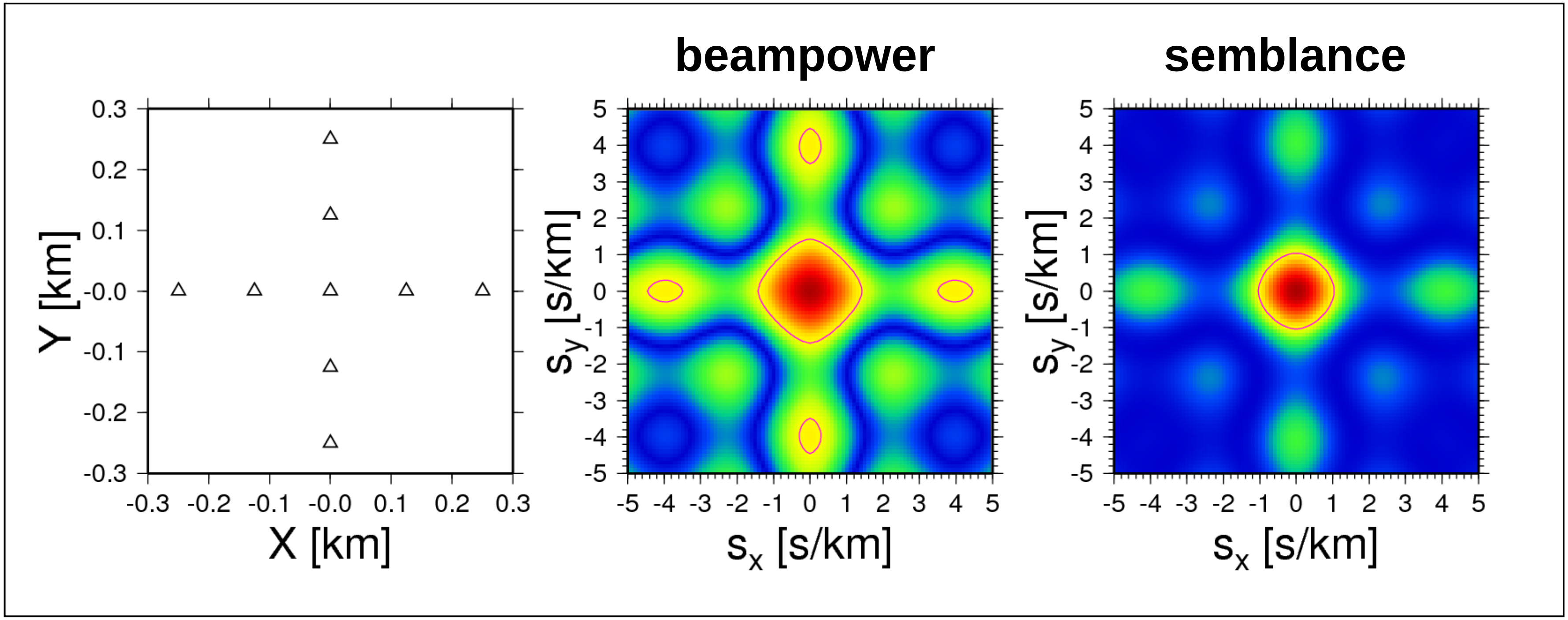
limits of array geometry: $\lambda_{min} > 2d_{min}$, $\lambda_{max} \sim 3D_{max}$

Array response
Extension to 2D situation – planar arrays

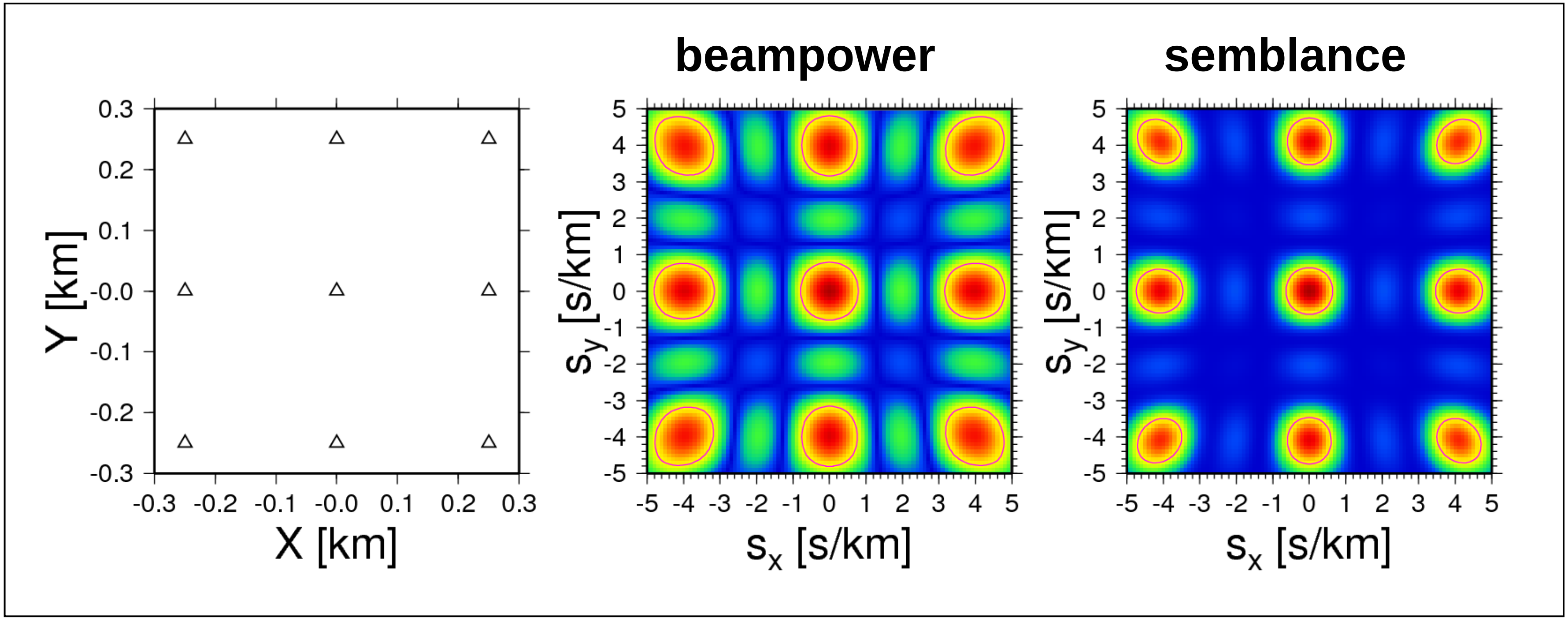
d_{min} and D_{max} show directional dependence



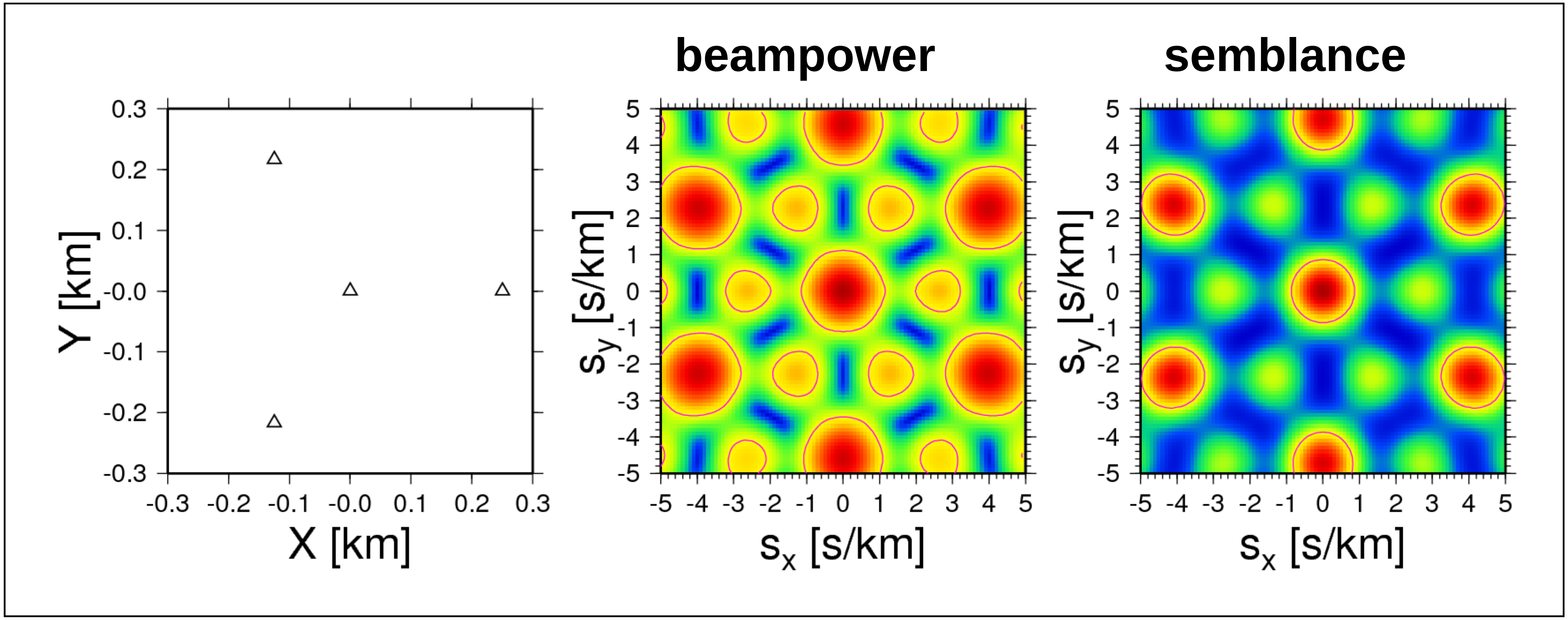
Some examples for typical symmetric array geometries



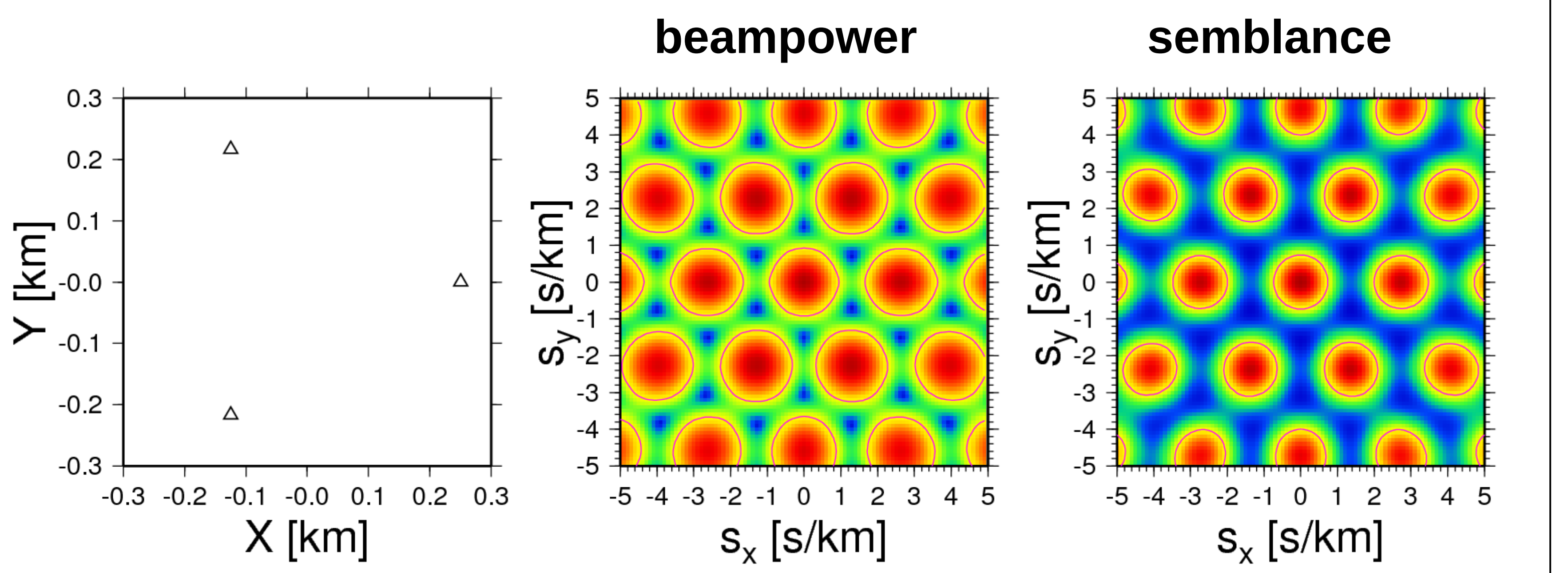
Some examples for typical symmetric array geometries



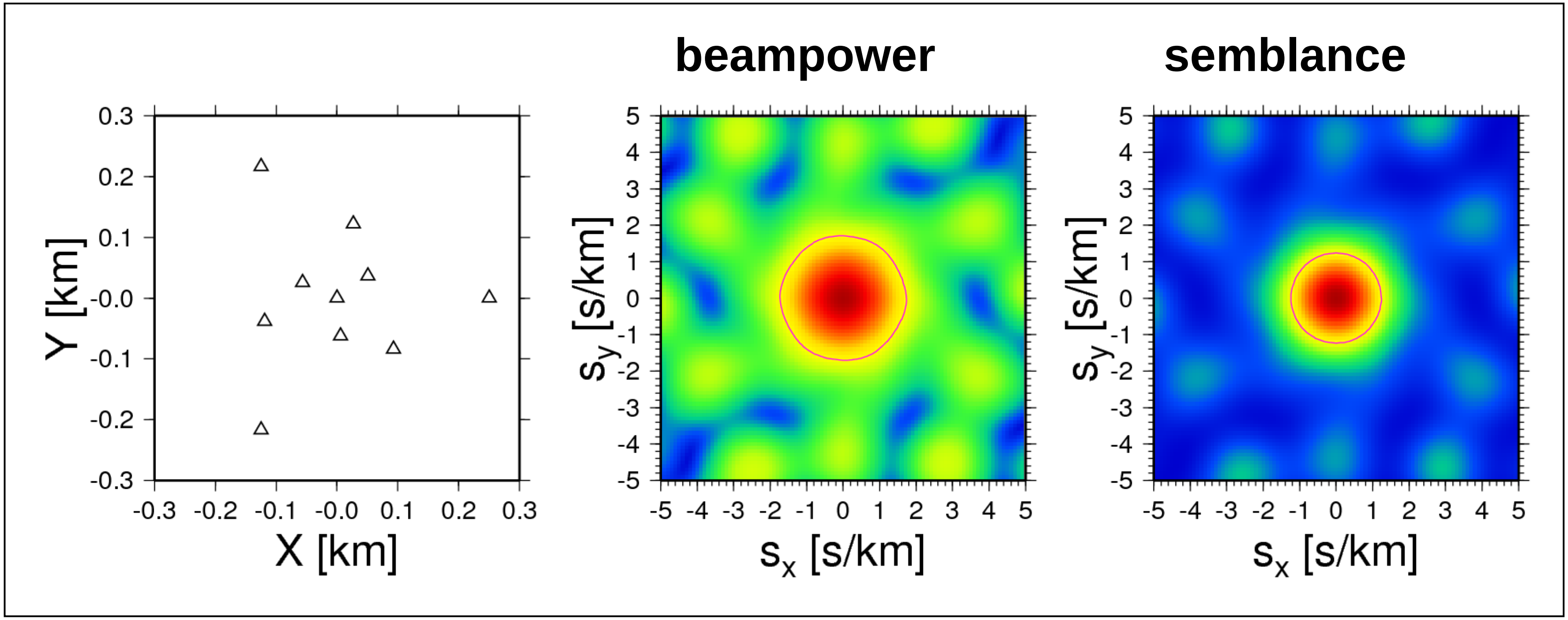
Some examples for typical symmetric array geometries



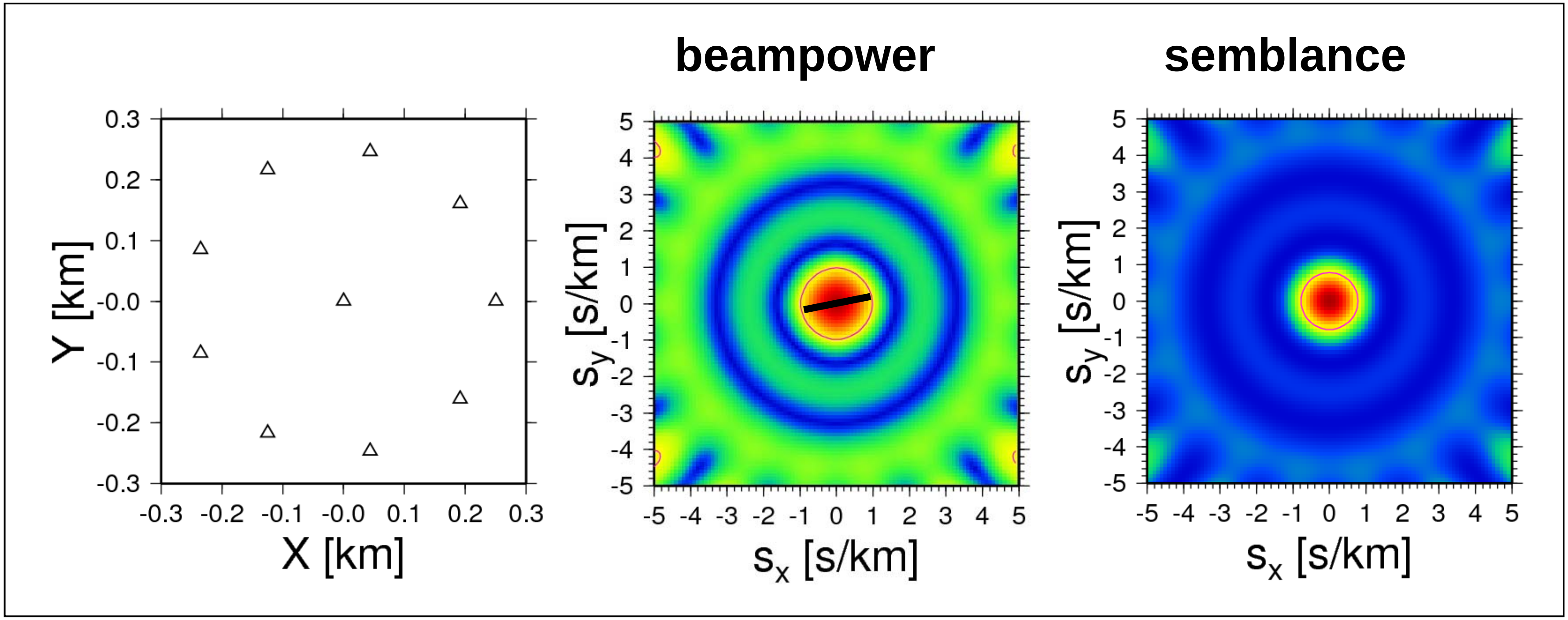
Some examples for typical symmetric array geometries



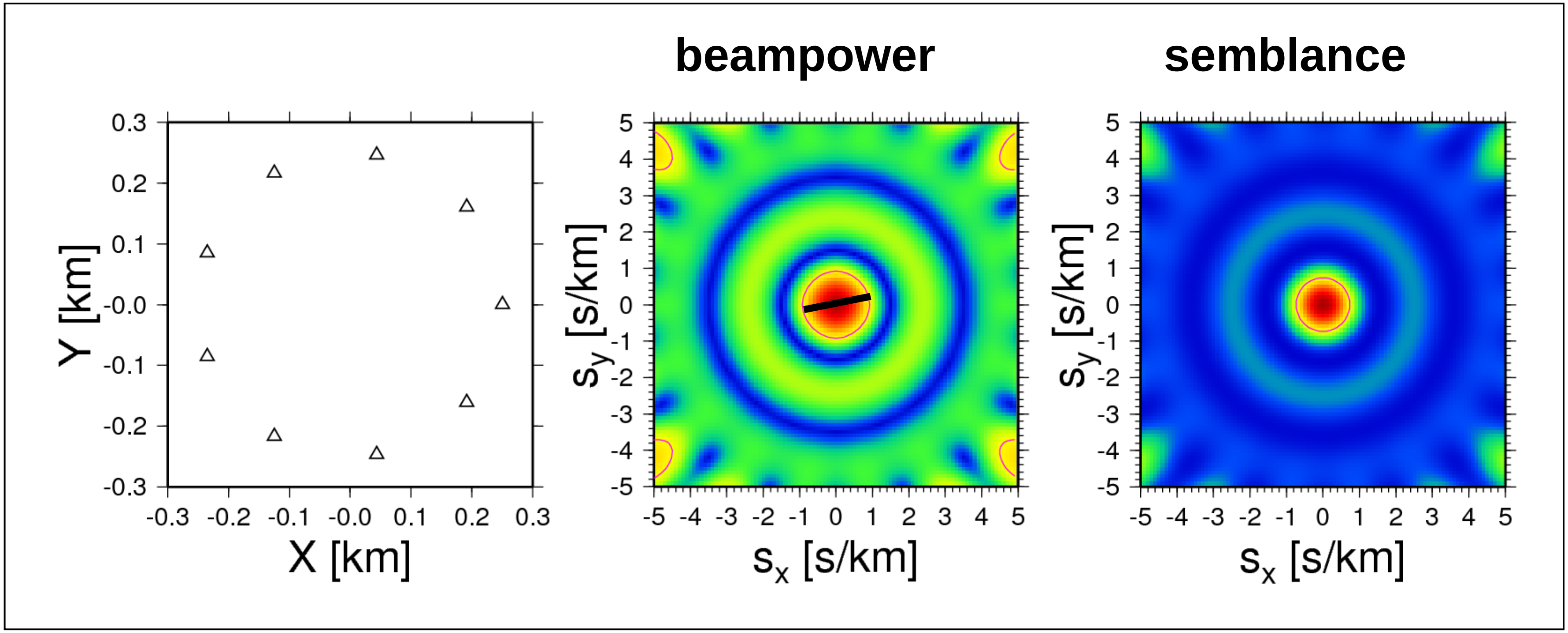
Some examples for typical symmetric array geometries



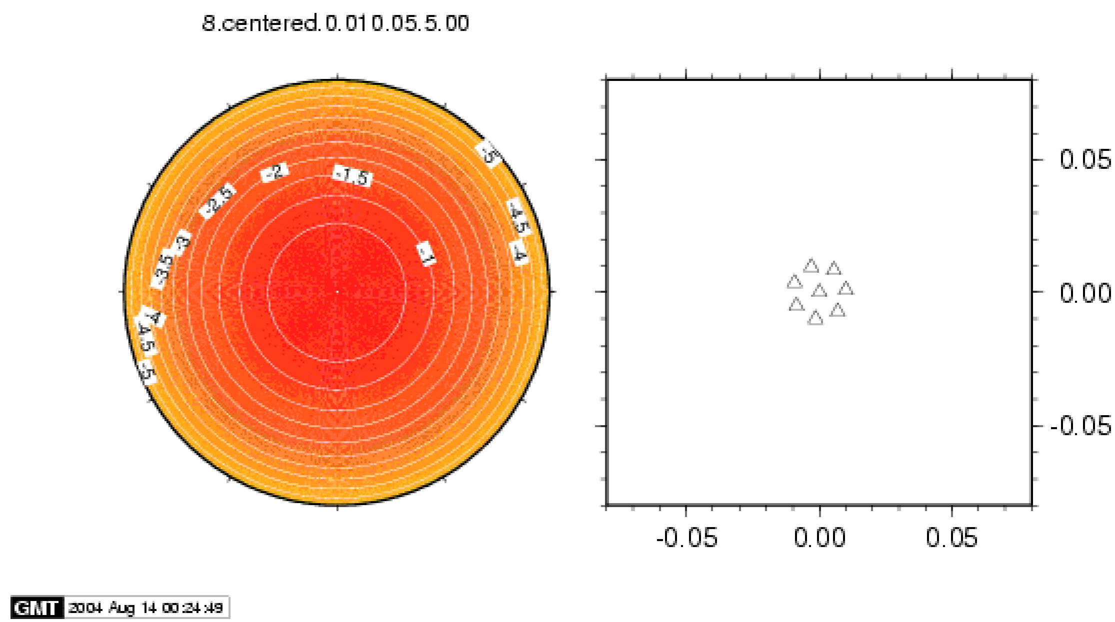
Some examples for typical symmetric array geometries



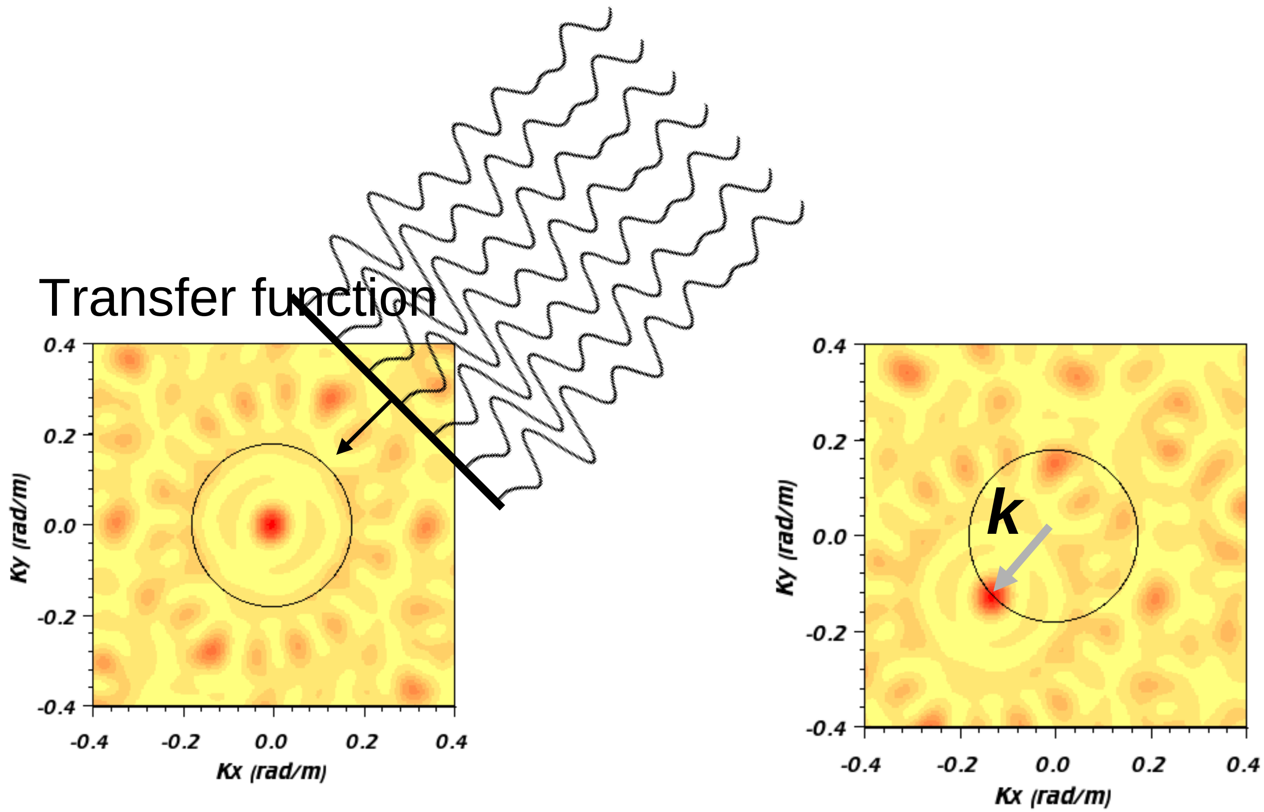
Some examples for typical symmetric array geometries



Some examples for typical symmetric array geometries



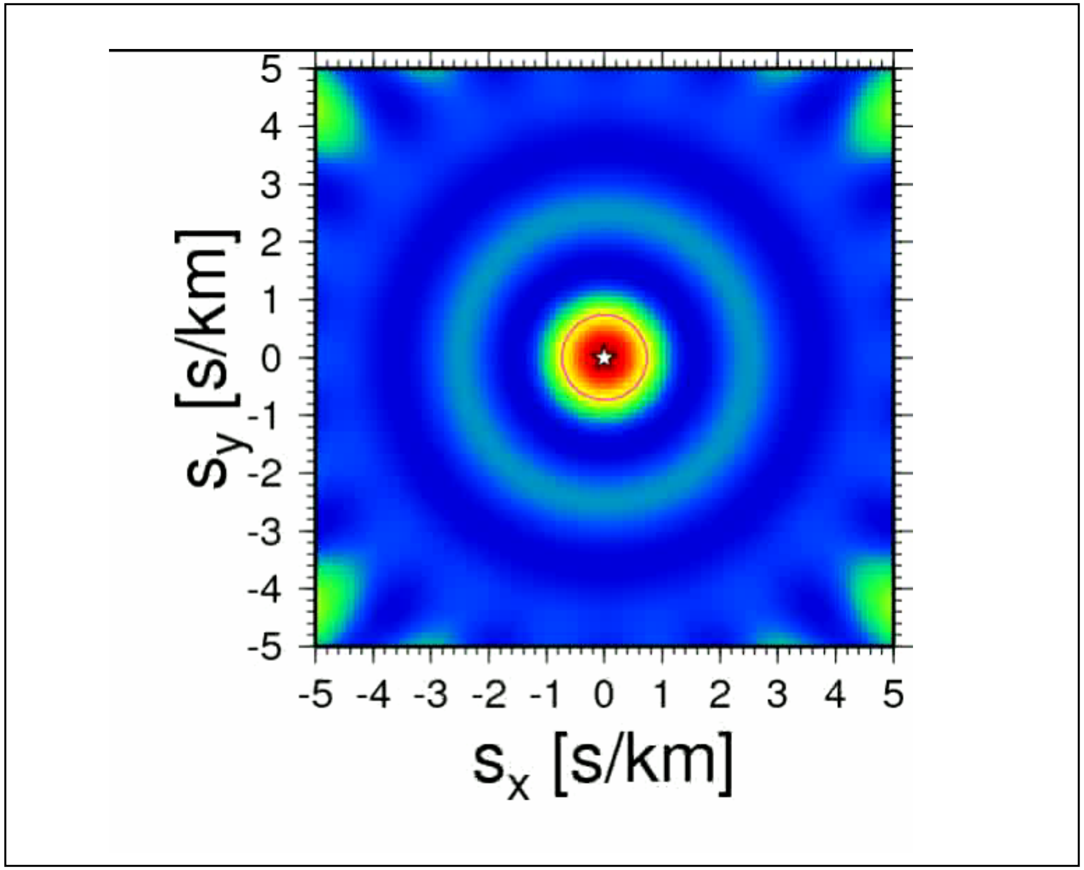
Array response moves with true slowness



Φ=atan(ky/kx)

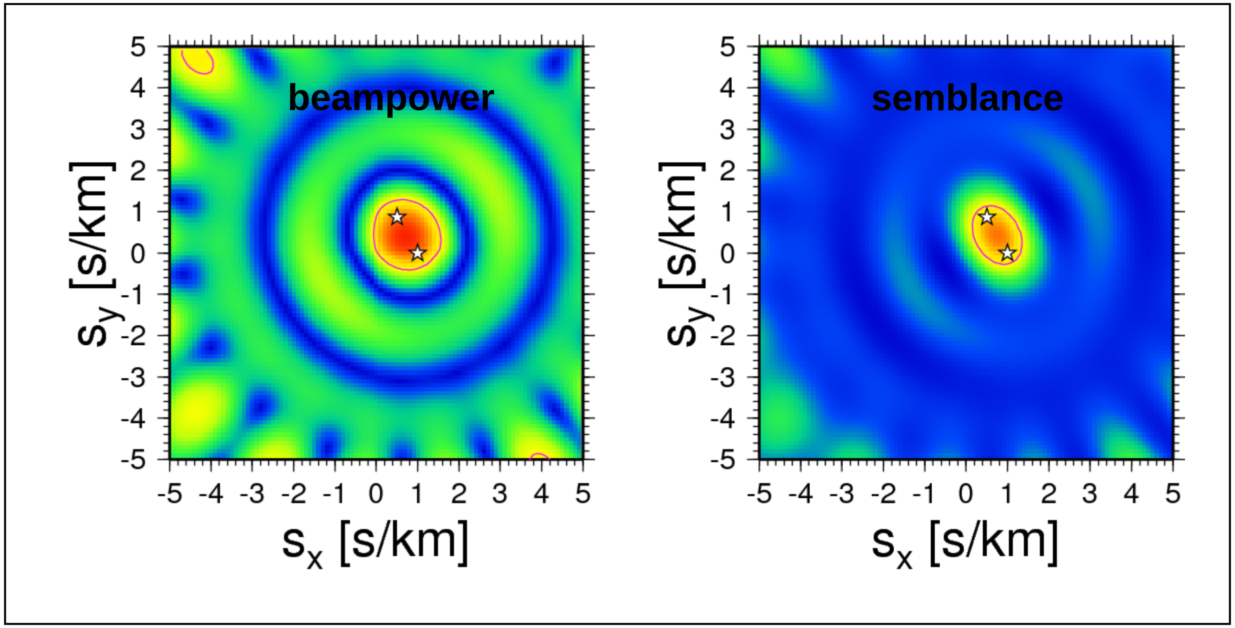
V=ω/|k|

Array response moves with true slowness



2D Array response – resolution limit?

2 sources, pure harmonic waves for [flow, fhigh] = [0.9,1.1] Hz

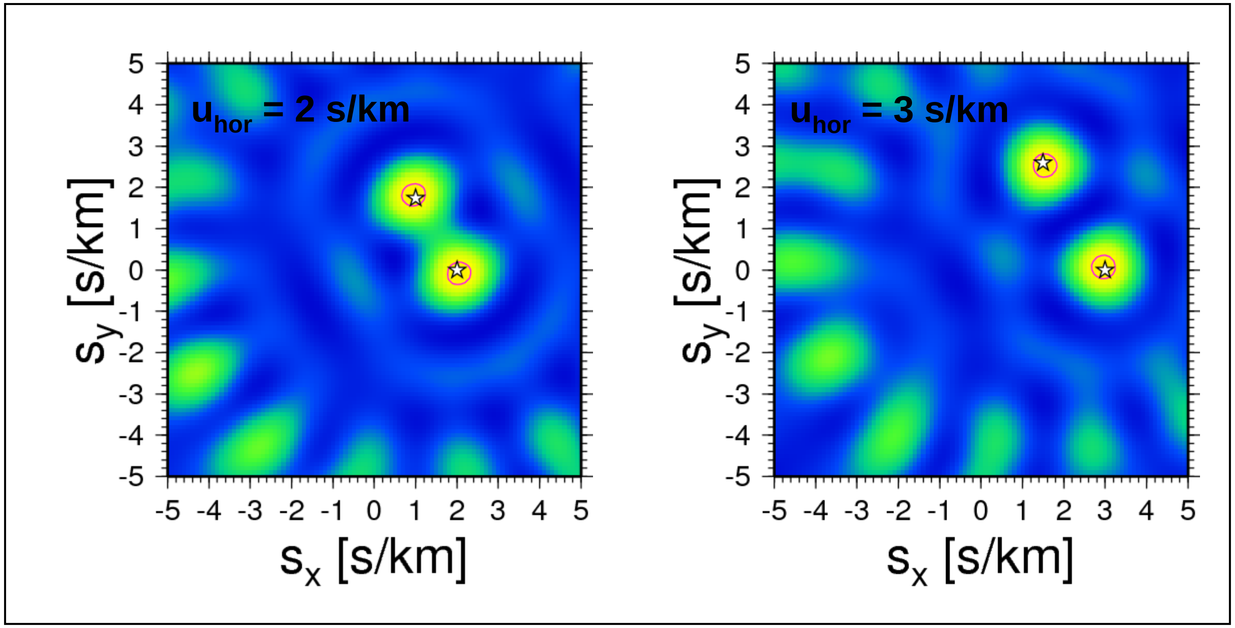


Waveparameter source A: u_{hor} = 1 s/km, θ = 90°

Waveparameter source B: u_{hor} = 1 s/km, θ = 30°

2D Array response – resolution limit?

2 sources, pure harmonic waves for [flow, fhigh] = [0.9,1.1] Hz



Waveparameter source A: θ = 90°

Waveparameter source B: θ = 30°

So far – so good – and now?

Now let‘s go finally to real life!

PURPOSE:

Using array techniques to analyze ambient vibration wavefields with the aim to derive shallow structural velocity models!

Background: Dispersion curve analysis

OVERVIEW: Array Analysis of Microtremor Wavefields
Applying basic principles (general) to a special problem domain

- What is special with microtremor wavefields?
- What is to be changed from the viewpoint of analysis?
- What is to be changed from the viewpoint of geometries?

→ Complications and attempts to deal with them

What is special with microtremor wavefields?

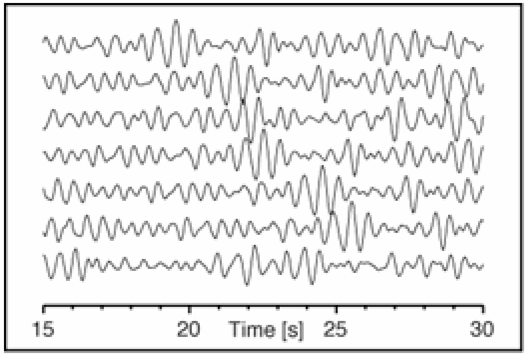
- **low energetic wavefield**
- **multiple sources (!)**
- **unknown (!) spatiotemporal structure of sources**
- **unknown (!) composition of wavefield, strong assumptions necessary**

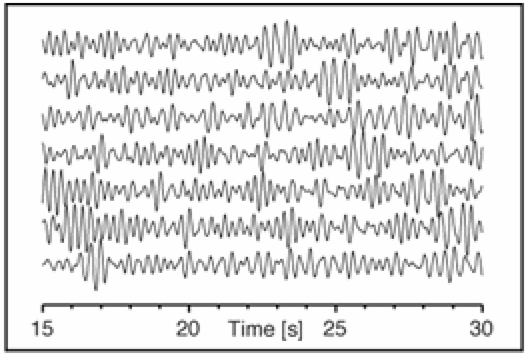
What is to be changed from the viewpoint of analysis?

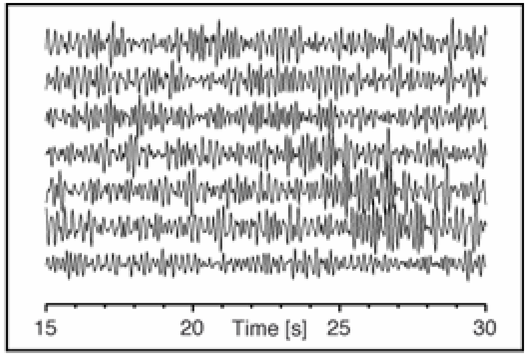
- adapt processing scheme for narrowband analysis of continuous data streams: processing of analysis windows with constant time-bandwidth product
- uncertainty estimate required:
  critical review of assumptions
- critical interpretation of results

What is to be changed from the viewpoint of analysis?

- **adapt processing scheme for narrowband analysis of continuous data streams: processing of analysis windows with constant time-bandwidth product**



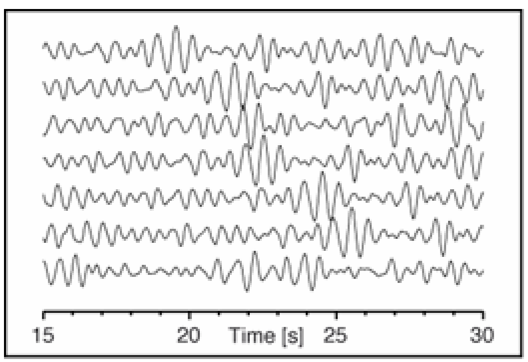


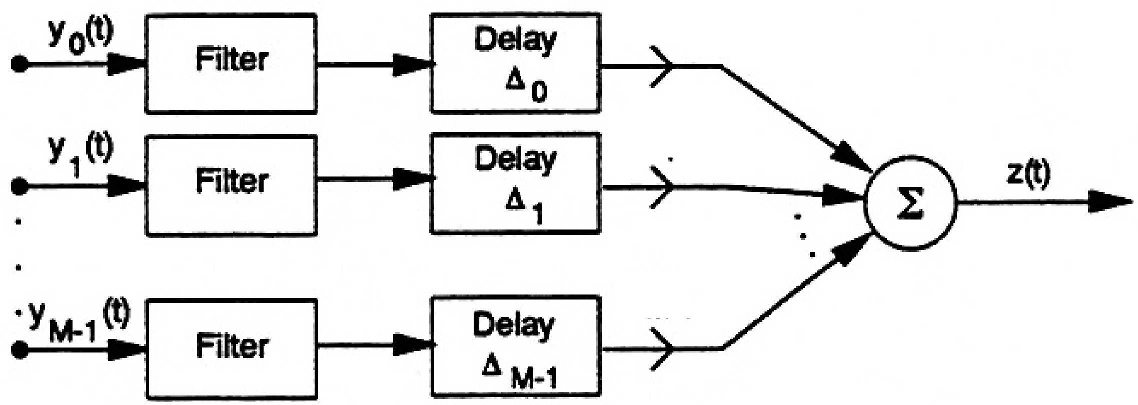


Bands:	1-2 Hz	2-4 Hz	4-8 Hz
fc:	1.5 Hz	3 Hz	6 Hz
bw:	1 Hz	2 Hz	8 Hz
Bw-frac:	33%	33%	33%
T:	66,6 s	33,3 s	16,6 s

What is to be changed from the viewpoint of analysis?

- **adapt processing scheme for narrowband analysis of continuous data streams: processing of analysis windows with constant time-bandwidth product**





Bands:	1-2 Hz	2-4 Hz	4-8 Hz
fc:	1.5 Hz	3 Hz	6 Hz
bw*T:	66,6	66,6	66,6

What is to be changed from the viewpoint of analysis?

- **uncertainty estimate required: critical review of assumptions**
- **critical interpretation of results**

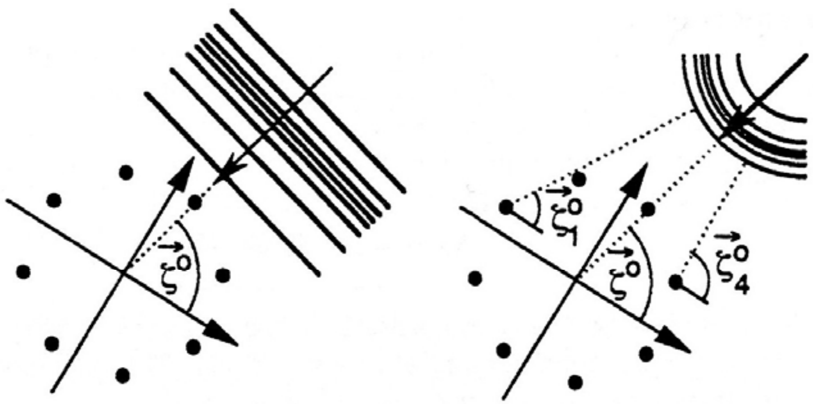
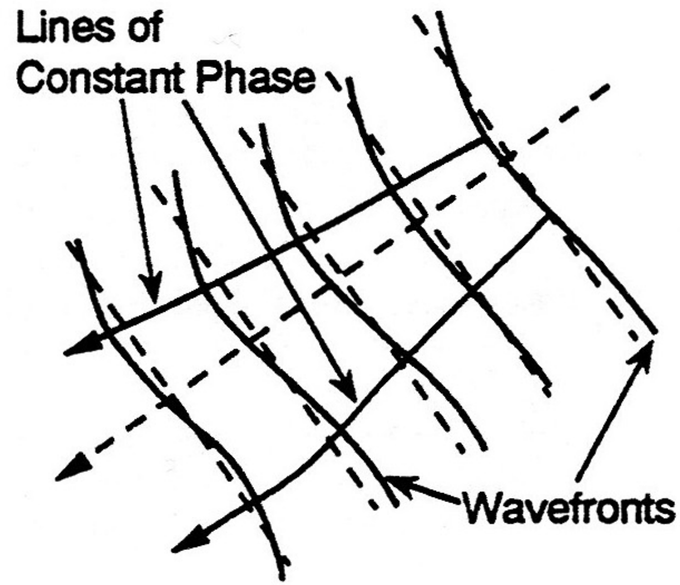


Figure 4.3 In near-field geometry, the source's angle with respect to the array's phase differs from those measured with respect to each sensor. The near-field–far-field discrepancy equals the angular difference between $\vec{\xi}^o$ and $\vec{\xi}^o_m$.

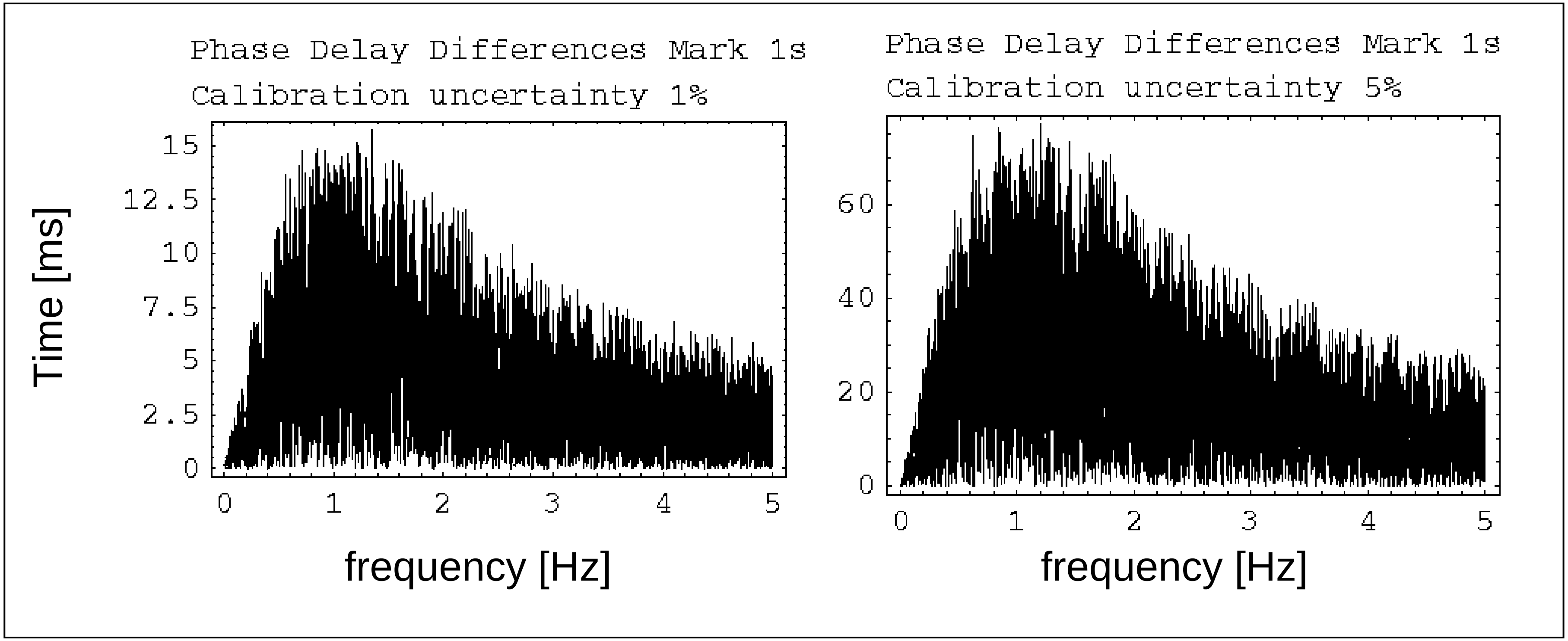


Plane wave assumption?

What is to be changed from the viewpoint of analysis?

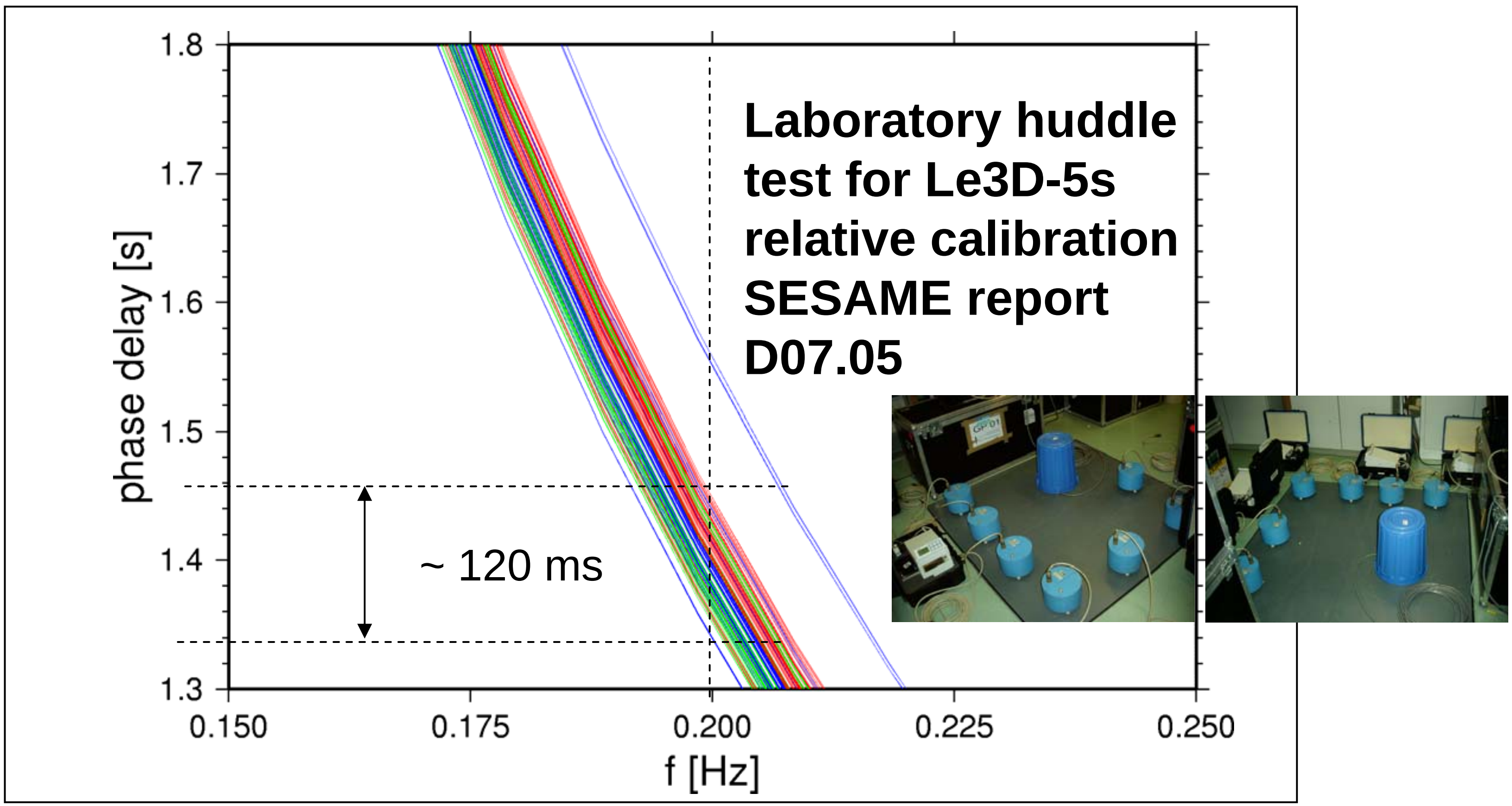
- **uncertainty estimate required: critical review of assumptions**

Instrumental effects? → Sensor stability / calibration



What is to be changed from the viewpoint of analysis?

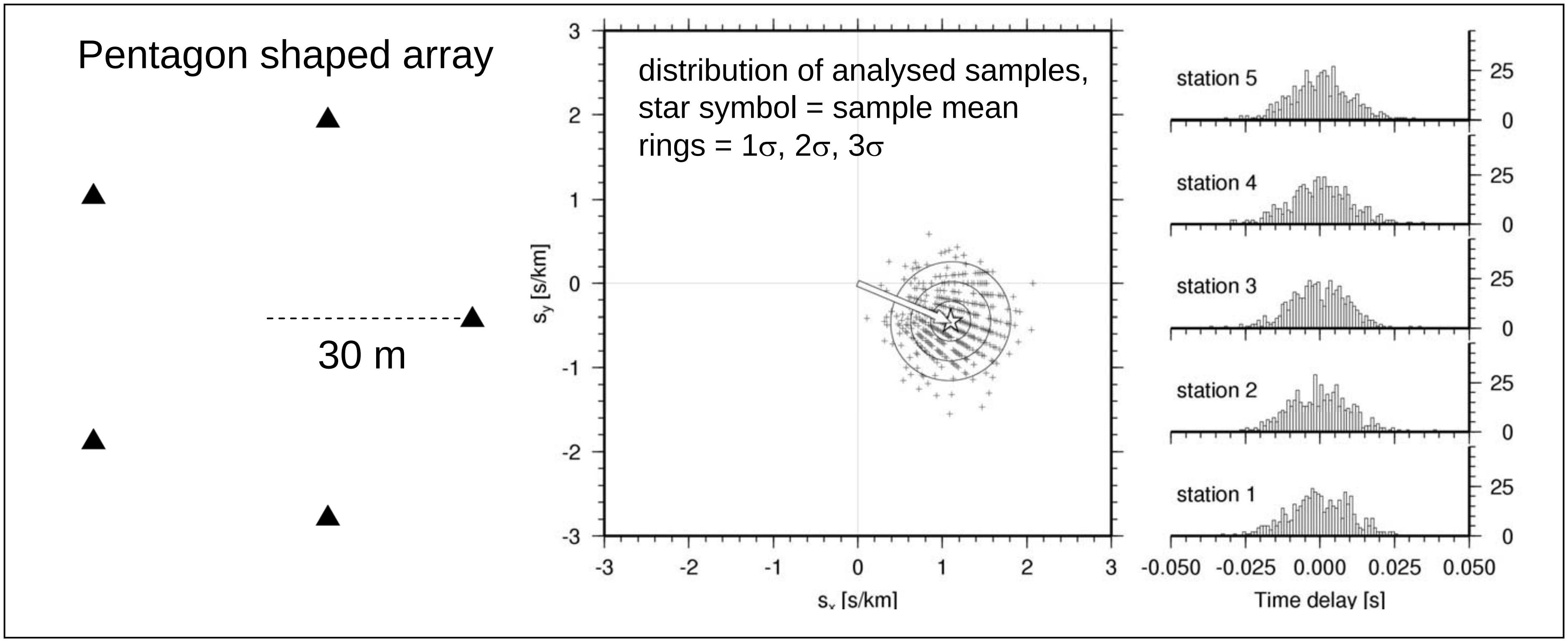
- **uncertainty estimate required:**
  critical review of assumptions



- **uncertainty estimate required: critical review of assumptions**

Quantification of random time delays on slowness estimates: a simple numerical bootstrap test

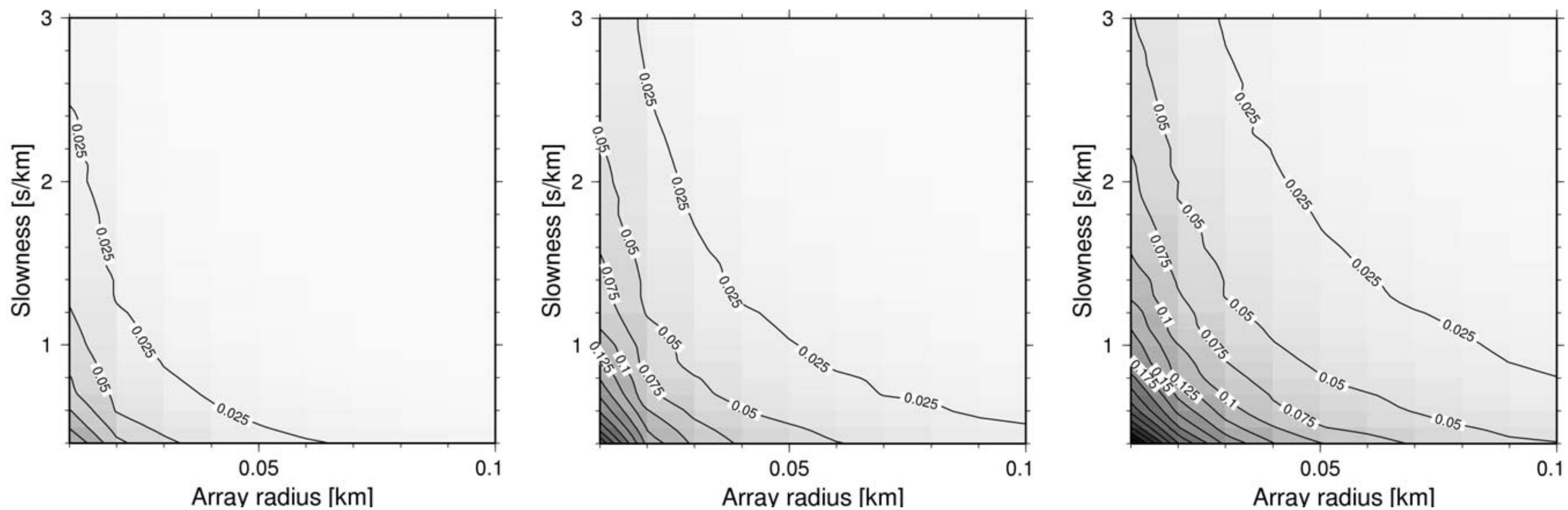
Example for 5 station array (pentagon shape) for array radius 30 m, Random time delays superimposed with μ=0 and σ=10 ms, 500 samples.



- **uncertainty estimate required: critical review of assumptions**

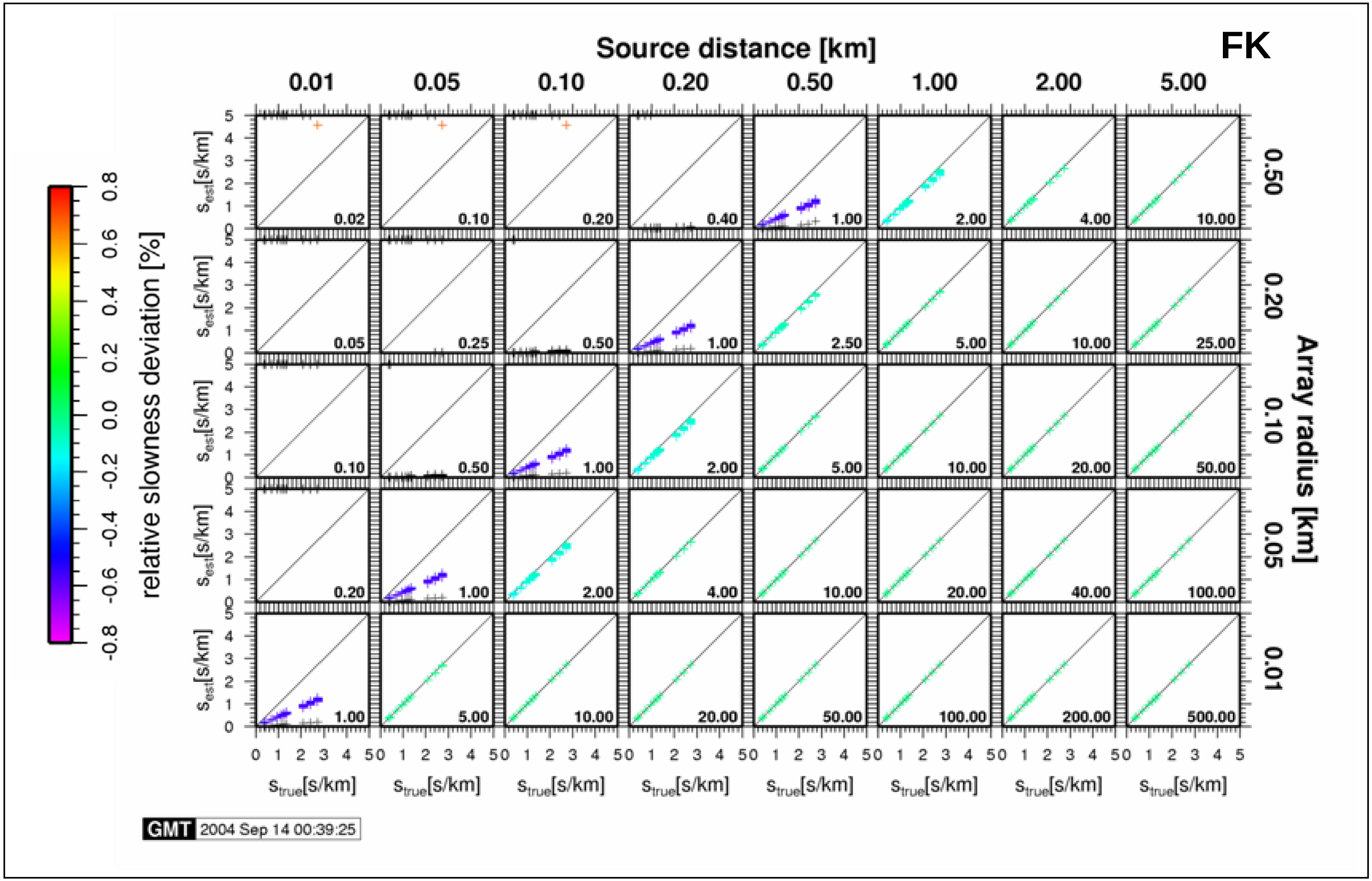
Quantification of random time delays on slowness estimates: a simple numerical bootstrap test

Example for 5 station array (pentagon shape) for different array radii, plane wave slownesses and variance of random time delays. Distribution for all stations: zero.mean, variance indicated by isolines.



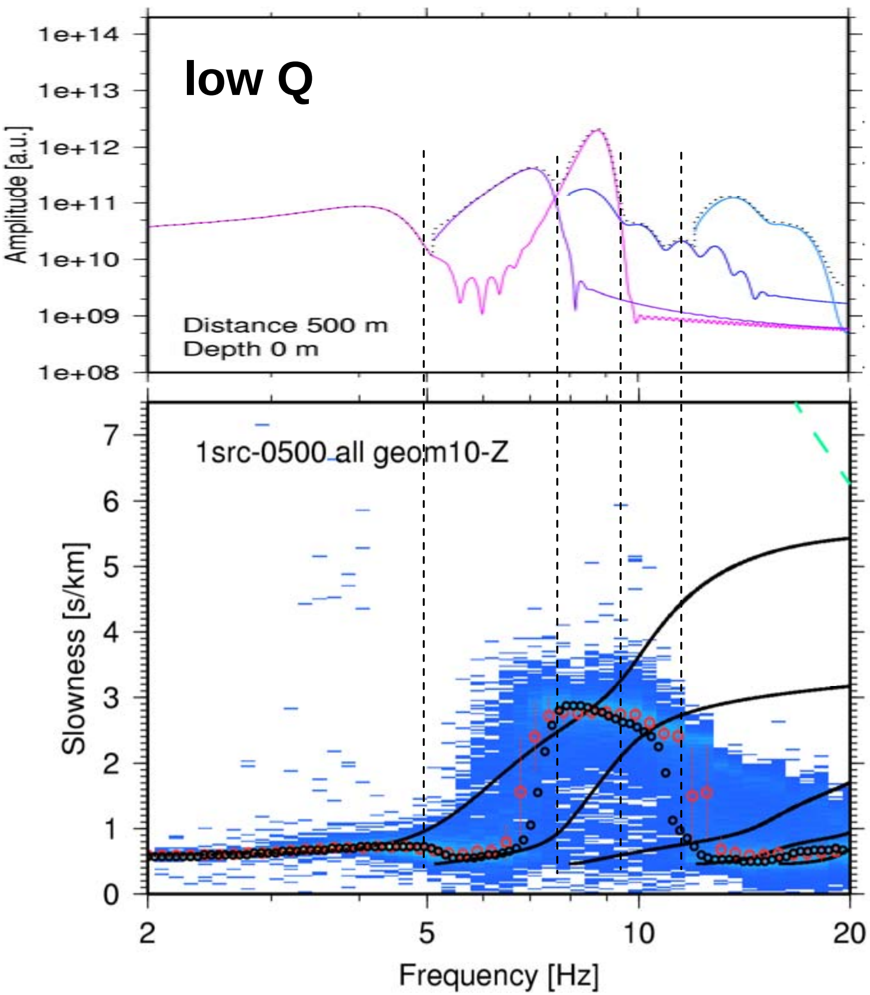
σ = 15 ms σ = 30 ms σ = 50 ms

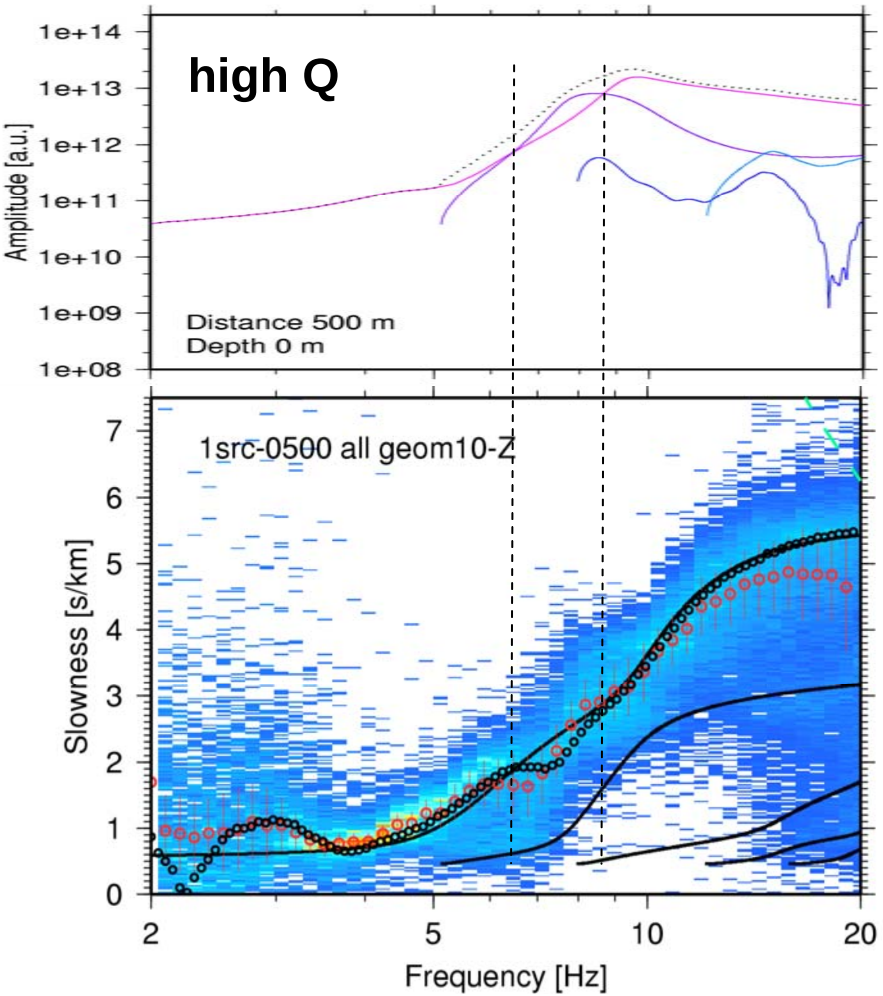
Violating the plane wave assumption …
what are the effects on the dispersion curve estimates?



What is to be changed from the viewpoint of analysis?

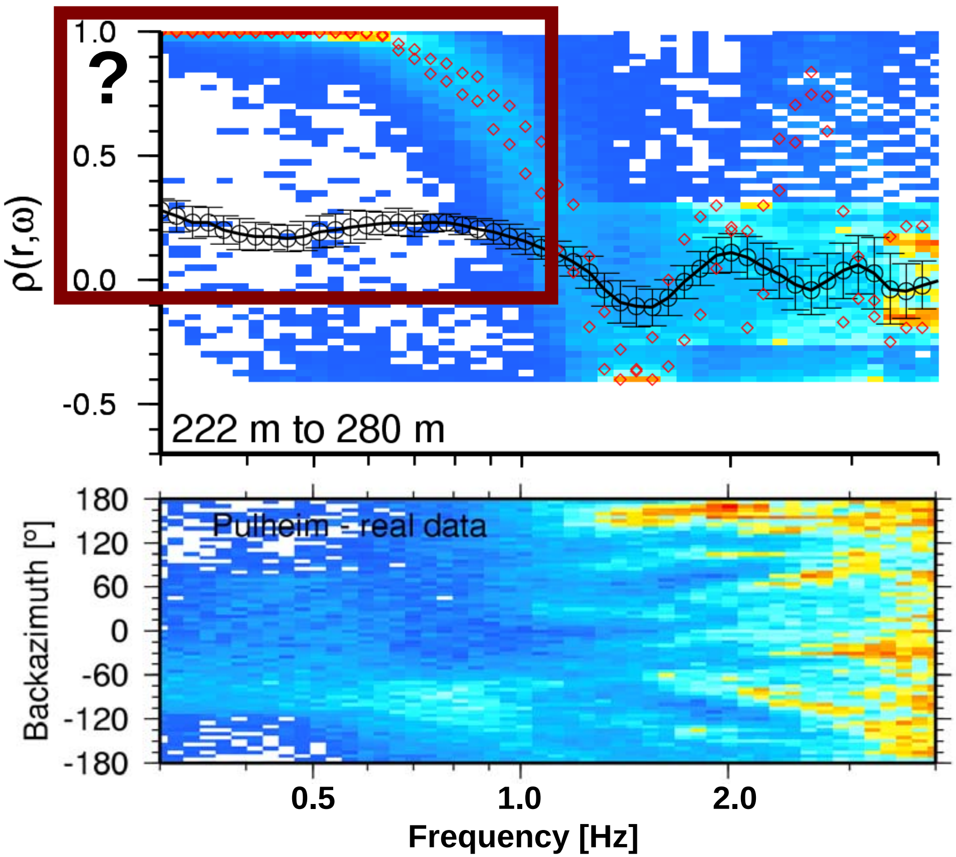
➢ **critical interpretation of results** Higher modes?

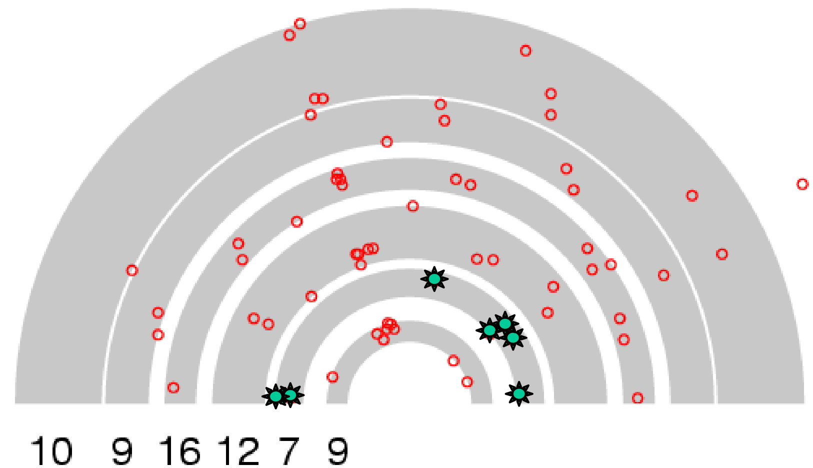




What is to be changed from the viewpoint of analysis?

➢ **critical interpretation of results** Method comparison!!

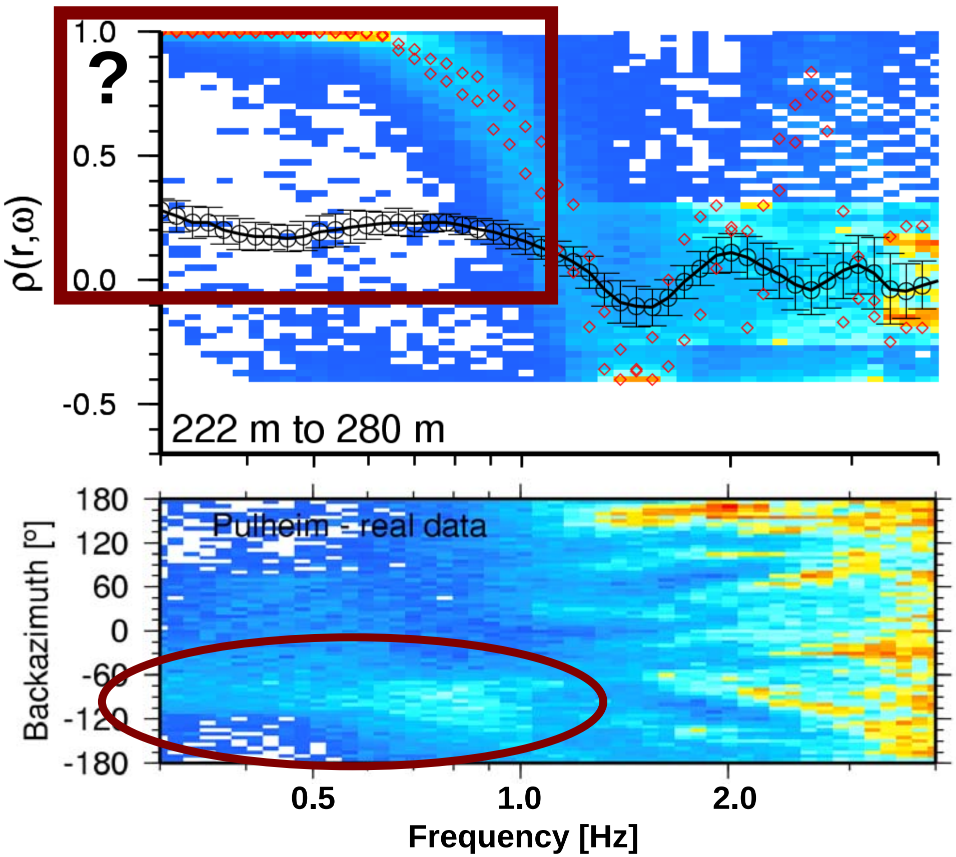


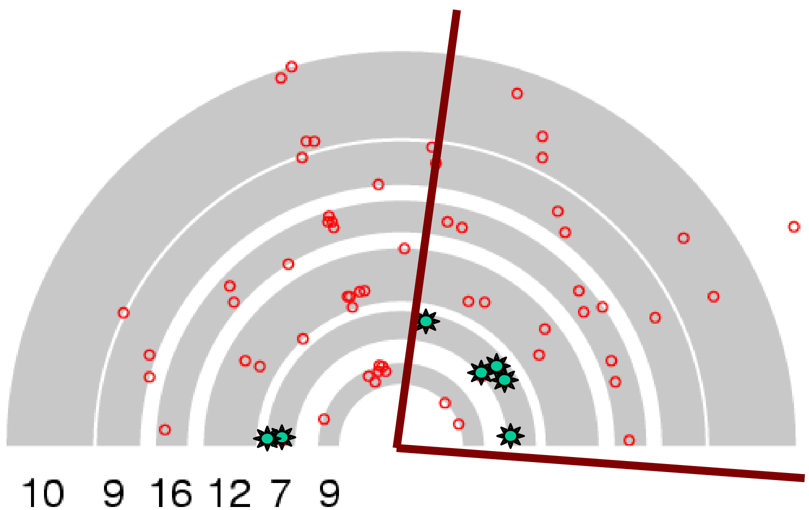


Interpretation:
Deviation between f-k and MSPAC is a result of insuffcent azimuthal sampling (difference between direction of wave propagation and interstation distance)

What is to be changed from the viewpoint of analysis?

- **critical interpretation of results** Method comparison!!





Interpretation:
Deviation between f-k and MSPAC is a result of insuffcent azimuthal sampling (difference between direction of wave propagation and interstation distance)

What is to be changed from the viewpoint of geometries?

- temporal experiments → N relatively low

- usually urban environment → logistical constraints

- no optimal single array configuration possible due to trade-off between number of stations, aperture and interstation distance

- existence of (very) local sources which violates the plane wave assumption → avoid wherever possible!

Derivation of dispersion curve - howto

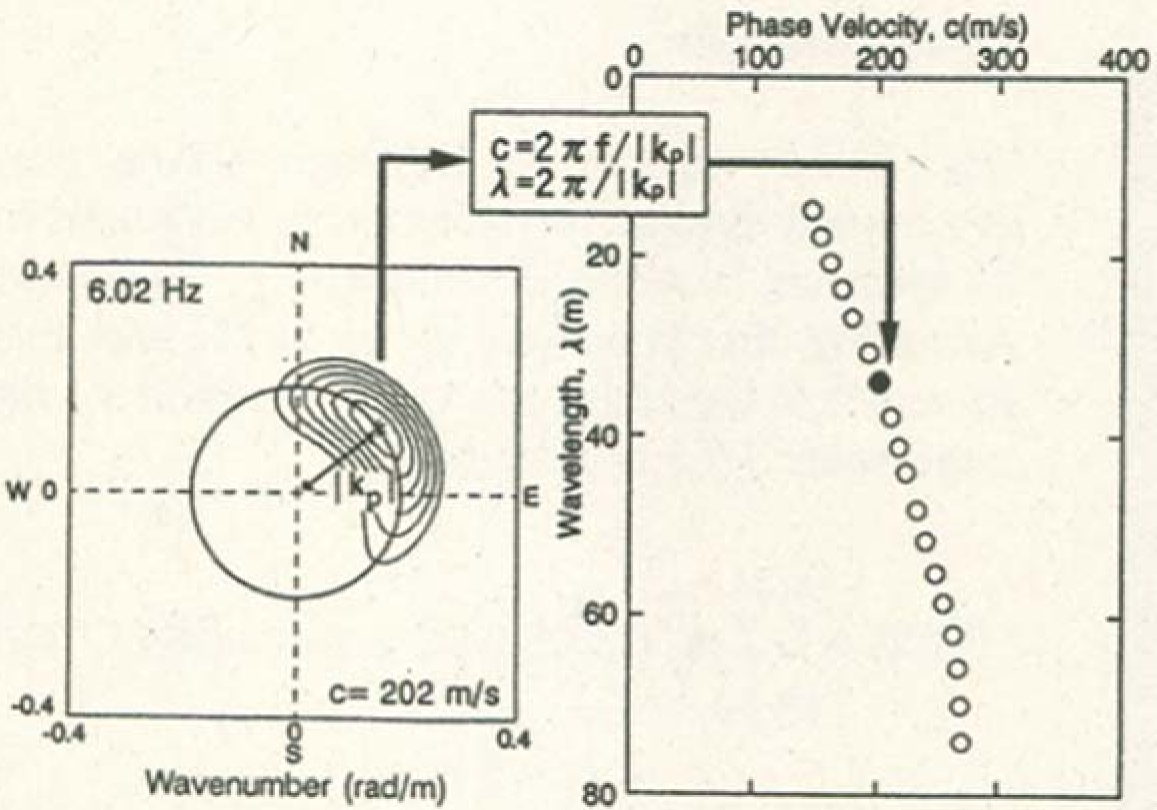
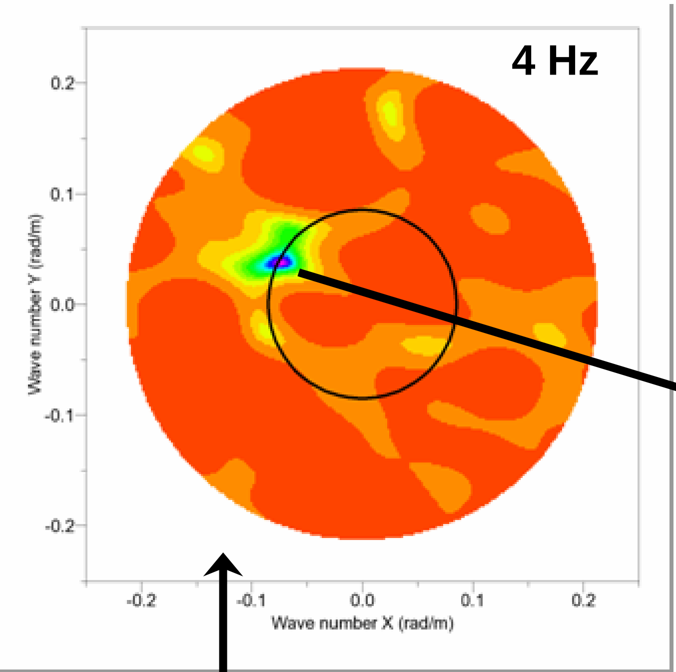


Fig. 29 A typical example of F–k spectra and corresponding dispersion data

e.g.:
* λ – v
* f – s
* f – v
* λ – s

from Tokimatsu, 1997

Geopsy preference → freq. vs. slowness



Single time window f-k analysis result; center frequency 4Hz bandwidth as fraction of center frequency