Using Ambient Vibration Array Techniques for Site Characterization

Surface Wave Inversion

Lecture

- What's an inverse problem?
- Inversion techniques
- Neighbourhood Algorithm (NA, Sambridge, 1999)
- Conditional parameter spaces
- Scaling, active models and discrete NA
- Inversion examples
- What can we do with inversion solution?

1. What's an inversion problem?

Parameters

Vp_0 Vs_0 H_0

Vp_1 Vs_1 H_1

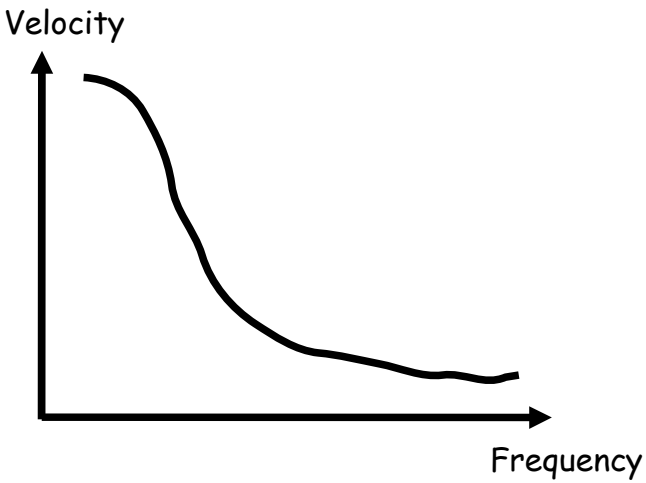
Vp_2 Vs_2

Ground model

Unknowns

Forward problem

Inverse problem

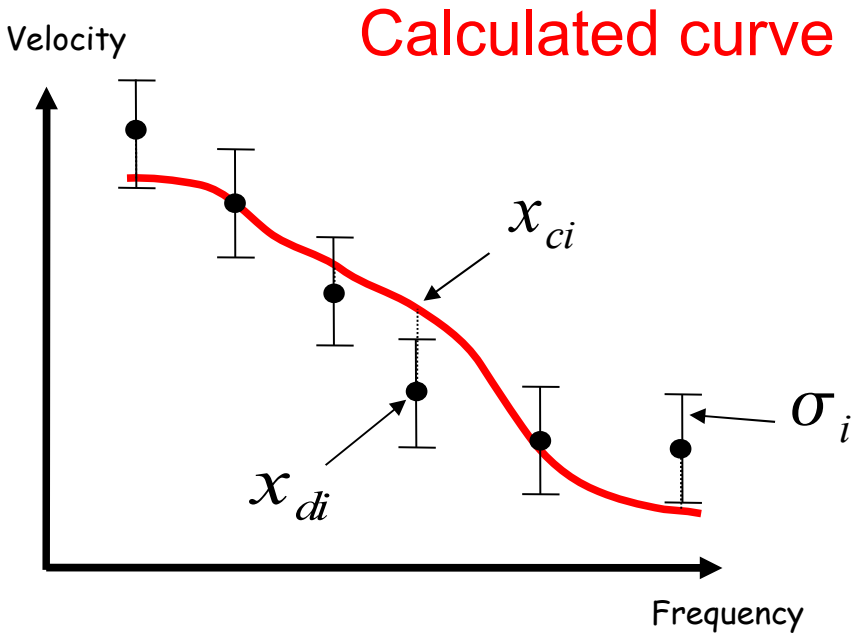


Dispersion curve or SPAC ratios

Data

Misfit definition

Ranking models
vs
Inversion target



$$\text{Misfit} = \sqrt{\sum_{i=1}^{n_F} \frac{(x_{di} - x_{ci})^2}{\sigma_i^2 \, n_F}}$$

n_F Number of frequency samples

A gentle 2D misfit function...

Imagine yourself without a map (nor a GPS) ...

... at the same place on a stormy day.

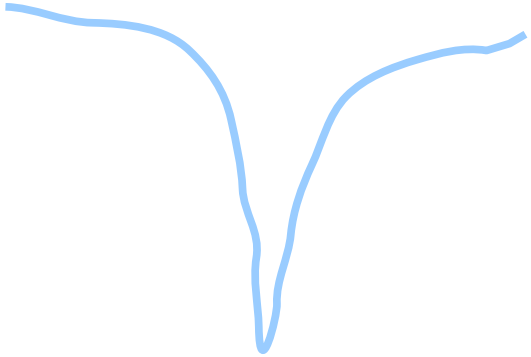
Where is the exit ? (= minimum misfit)

Where is the exit ?

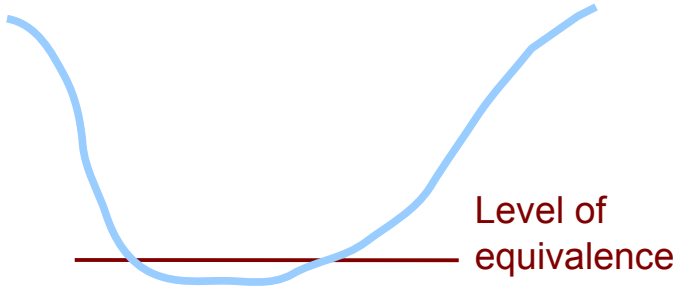
Start from anywhere and go down?

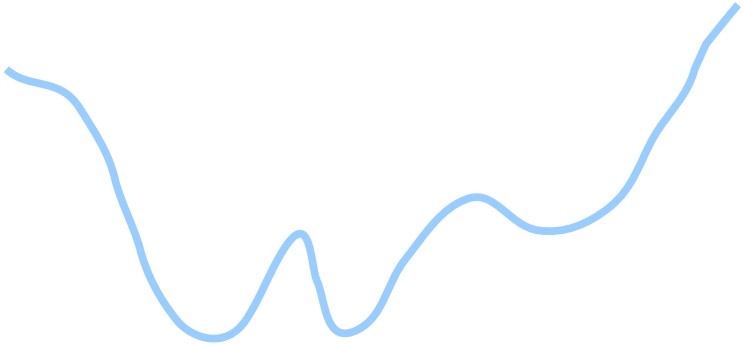
Possible shapes for a misfit function

A single narrow valley

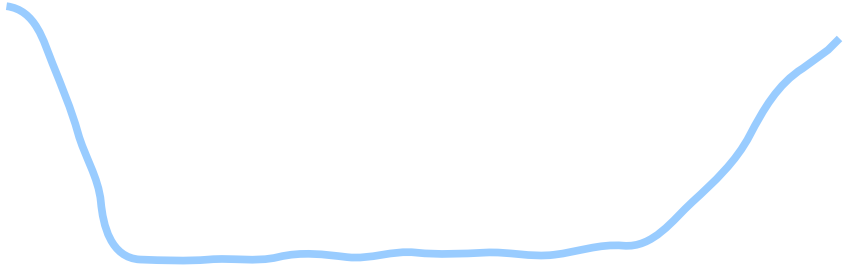


A larger valley





Local minima and
equivalent minima



A truly flat valley

2. Inversion Techniques

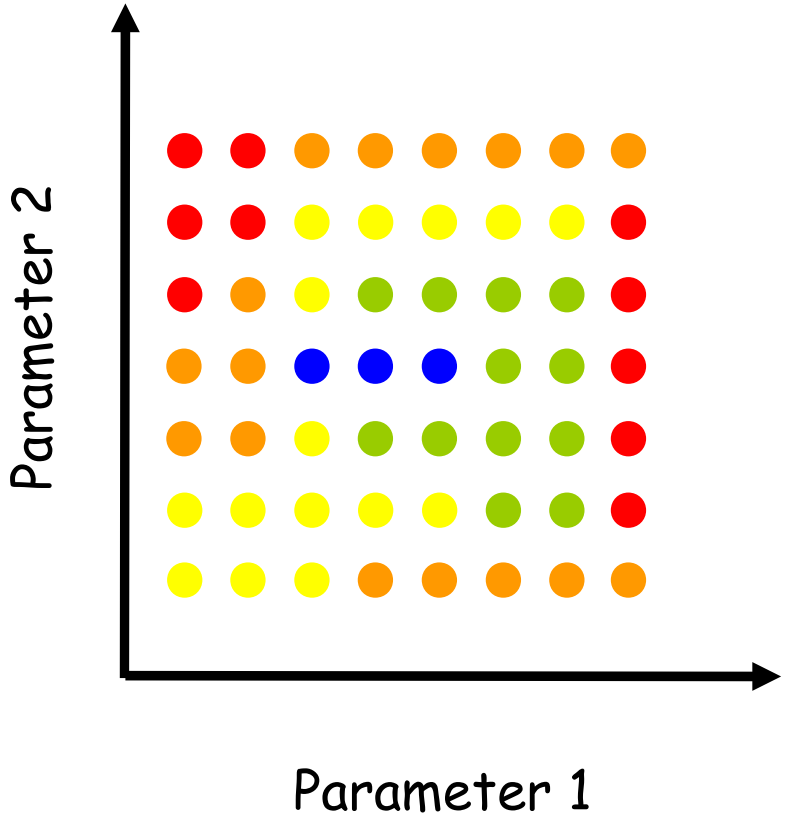
Forward problem:

- Analytic or numerical processing
- Only one solution

Inverse problem:

- Trial and error to adjust parameters of the model
- Simplex downhill method
- Brute force uniform search (gridding)
- Least square methods (based on derivatives)
- Brute force Monte Carlo sampling
- Simulated Annealing
- Genetic Algorithm
- Neighbourhood Algorithm
- Generally not only one solution

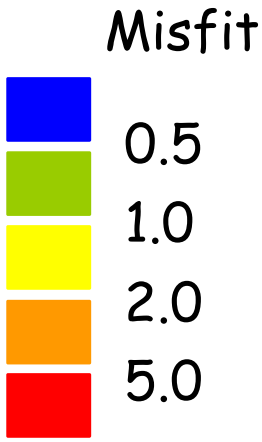
A. Uniform search (gridding)



- If nd > 3 : number of forward computations are prohibitive

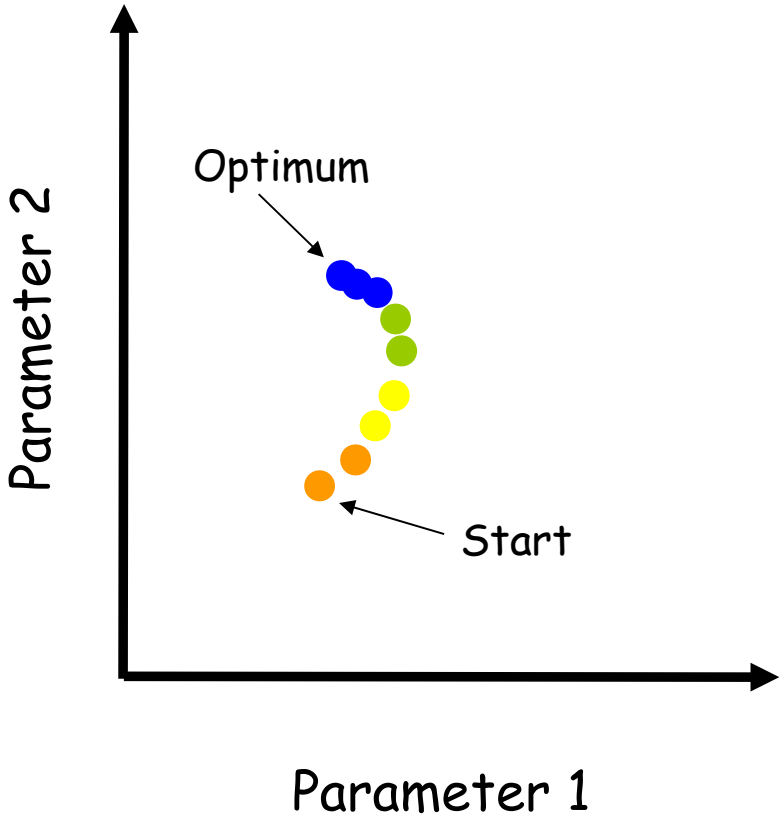
+ Complete exploration of the parameter space

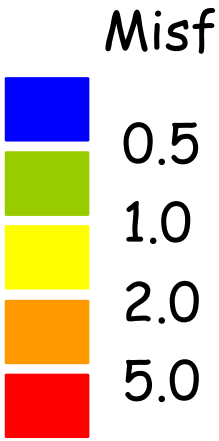
+ Optimum error estimates



B. Iterative methods (optimisation)

Least Square, Simplex, Gradient methods, ...

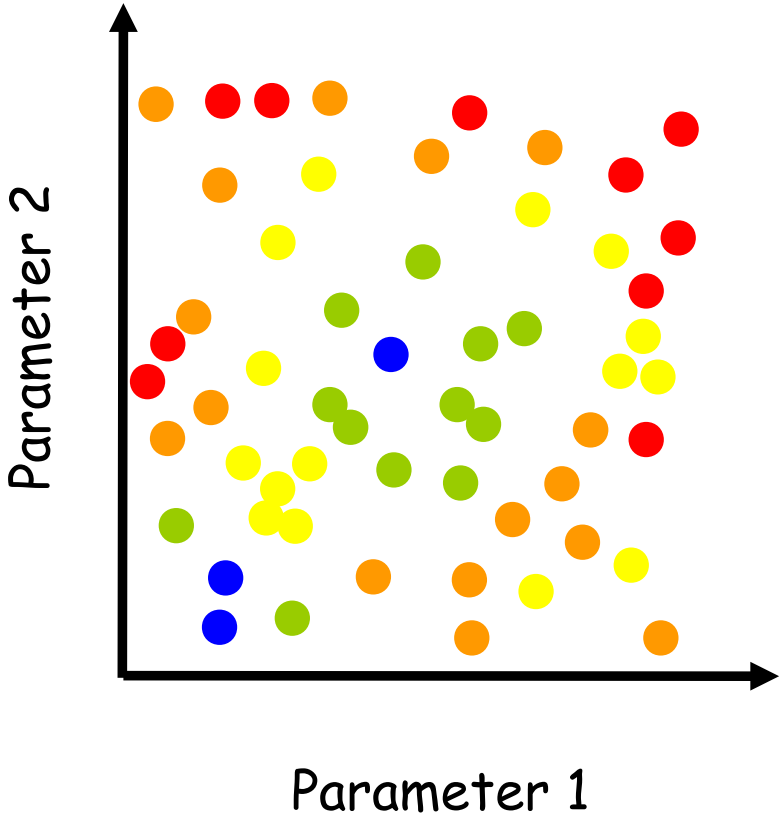




- Easily trapped in local minima
- Non-uniqueness <=> choice of starting model
- Bad error estimates
- Cannot include prior information

+ High dimensionality
+ Few forward computations

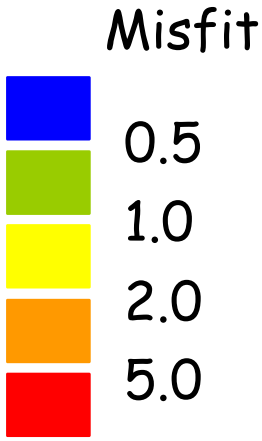
C. Random search (Monte Carlo)



- Requires lot of forward computations

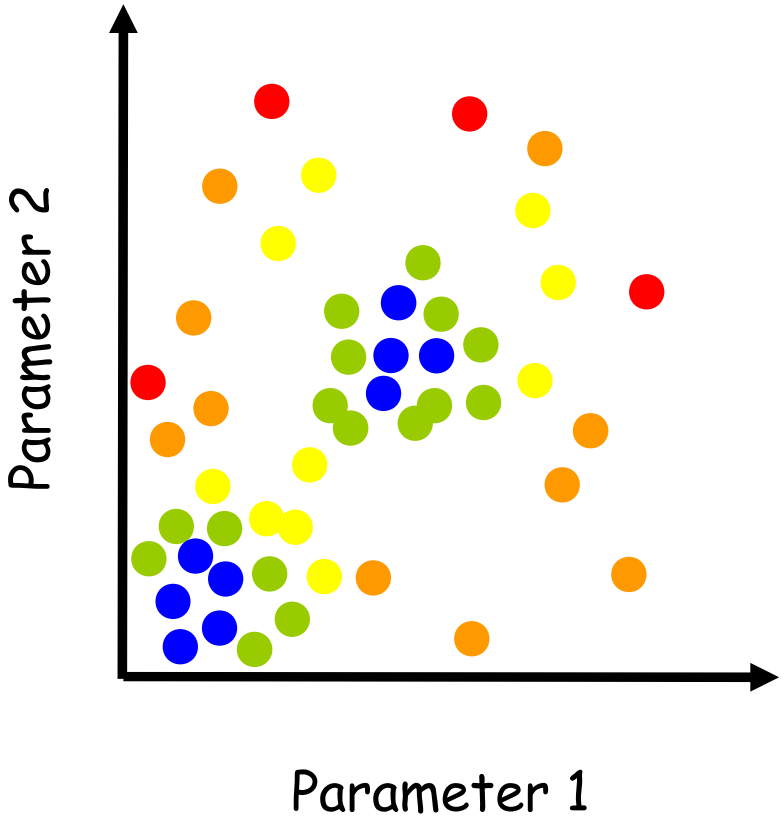
+ Not too bad exploration of the parameter space

+ Good error estimates

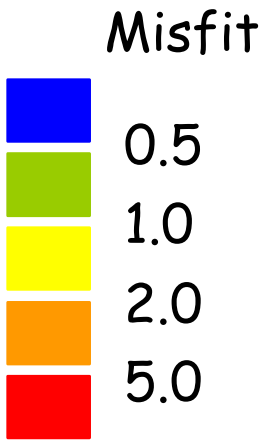


D. Oriented random search (~ 1990)

(SA, GA, and NA)

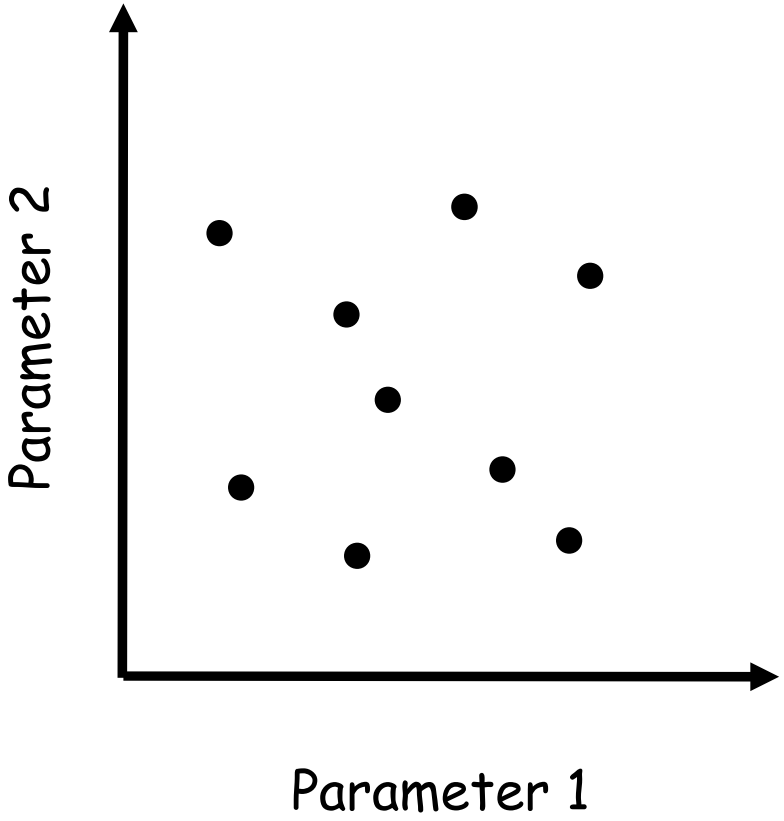


+ Requires less of forward computations than MC
- Max nd ~ 25-50
+ Not too bad exploration of the parameter space
+ Good error estimates



3. Neighborhood Algorithm

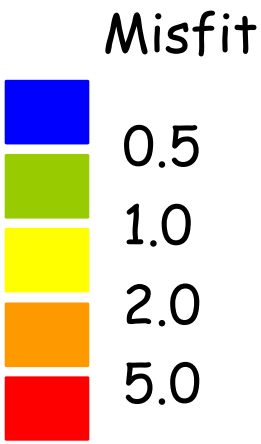
(Sambridge, 1999)



Few tuning parameters (Ns, Nr)

Based on Voronoi division of the parameter space

~ SA and GA (better according to Sambridge)



Ns new samples generated into Nr selected cells

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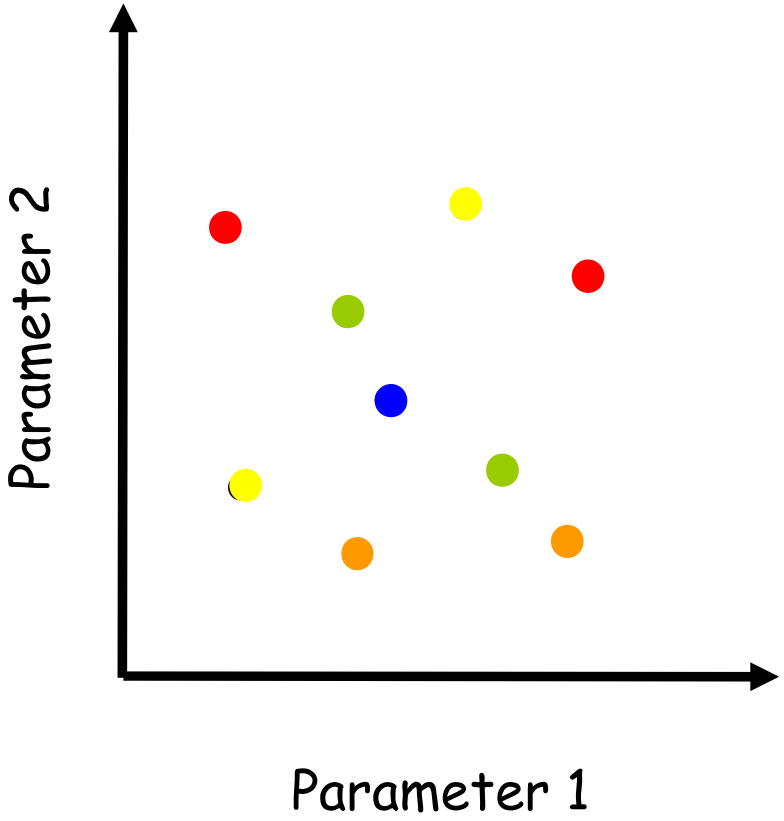

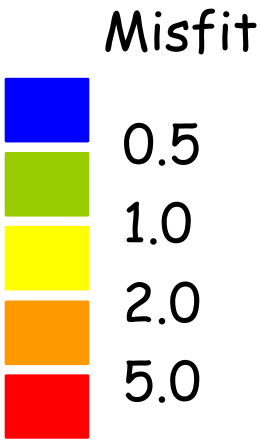

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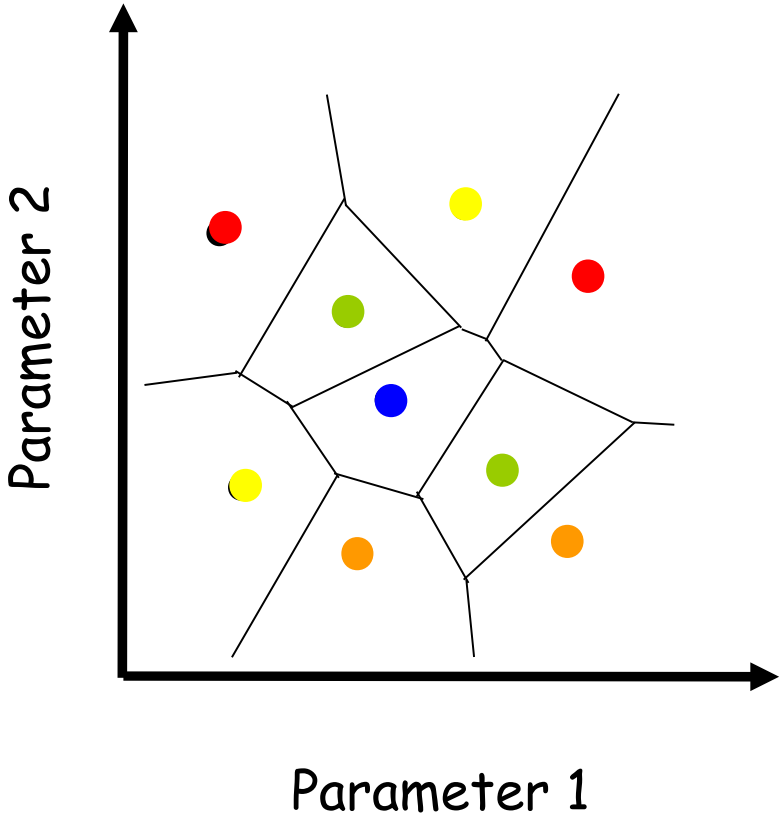

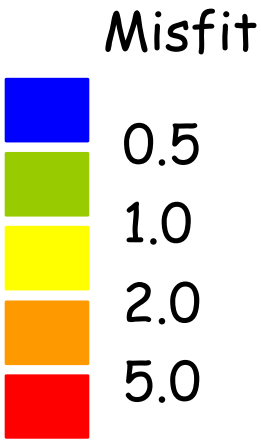

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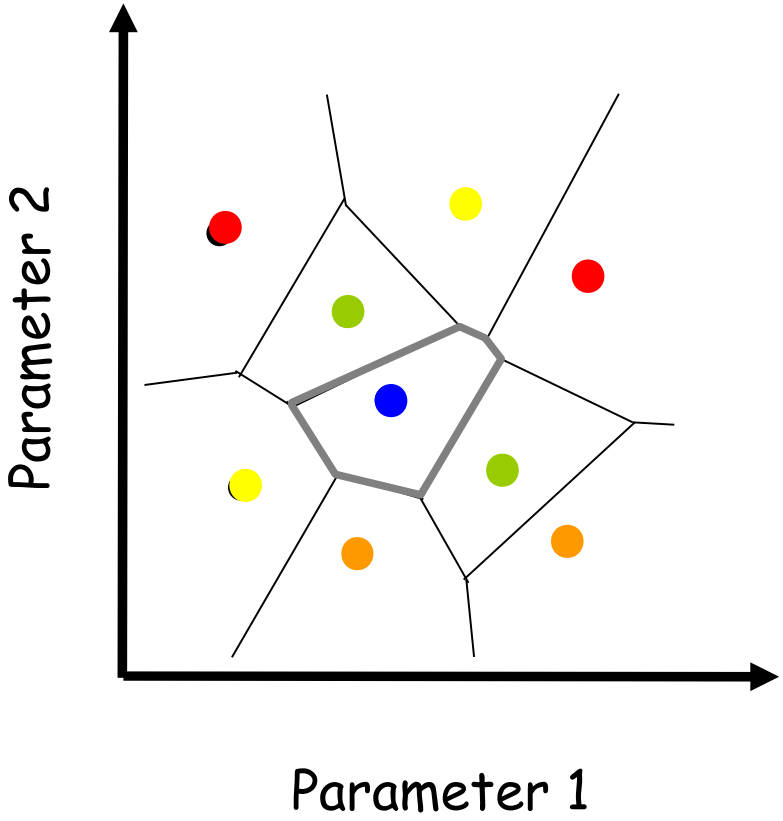

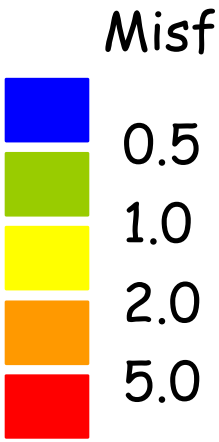

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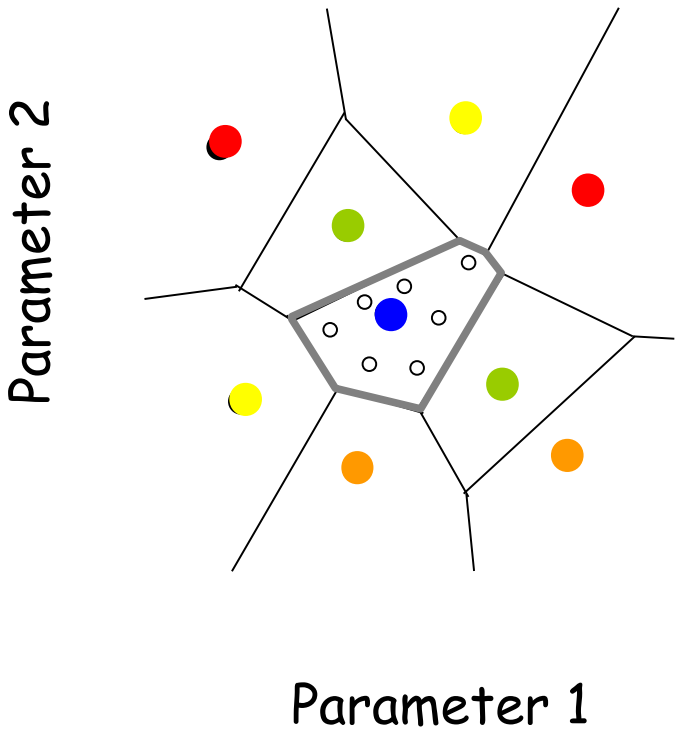


Misfit

0.5

1.0

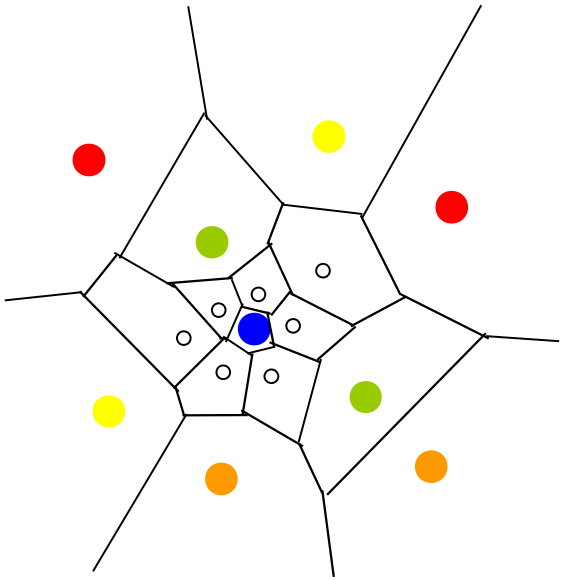
2.0

5.0

Ns new samples generated into Nr selected cells

3. Neighborhood Algorithm

(Sambridge, 1999)



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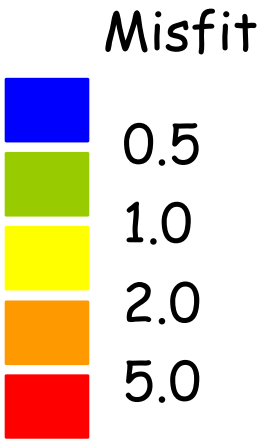


Ns new samples generated into Nr selected cells

NA for dispersion curves (DC) inversion

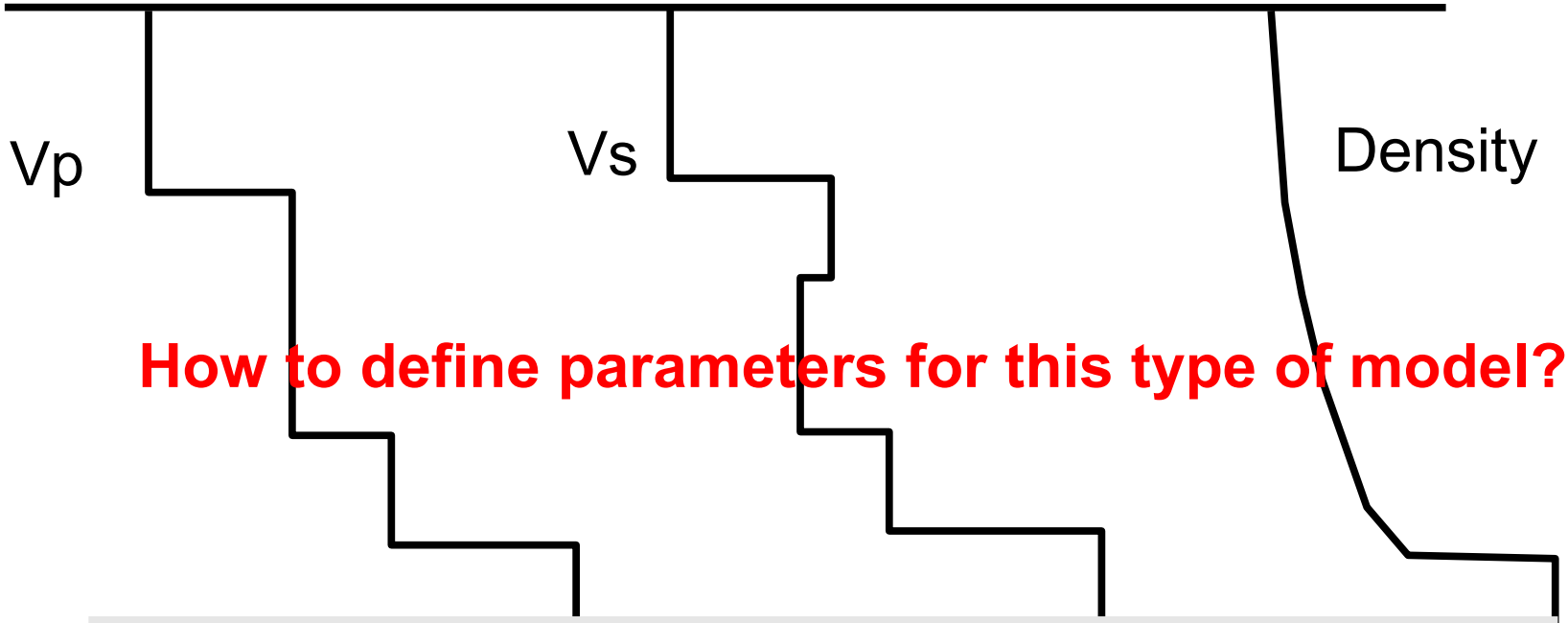
High number of forward computation required (~50,000)

Computation of DC for 1D elastic model =
numerical process not always stable

(Wathelet, 2005)

==> Battery of tests to automatically tune computation
==> Improvement of algorithm efficiency (~ ms/model)

4. Conditional neighborhood

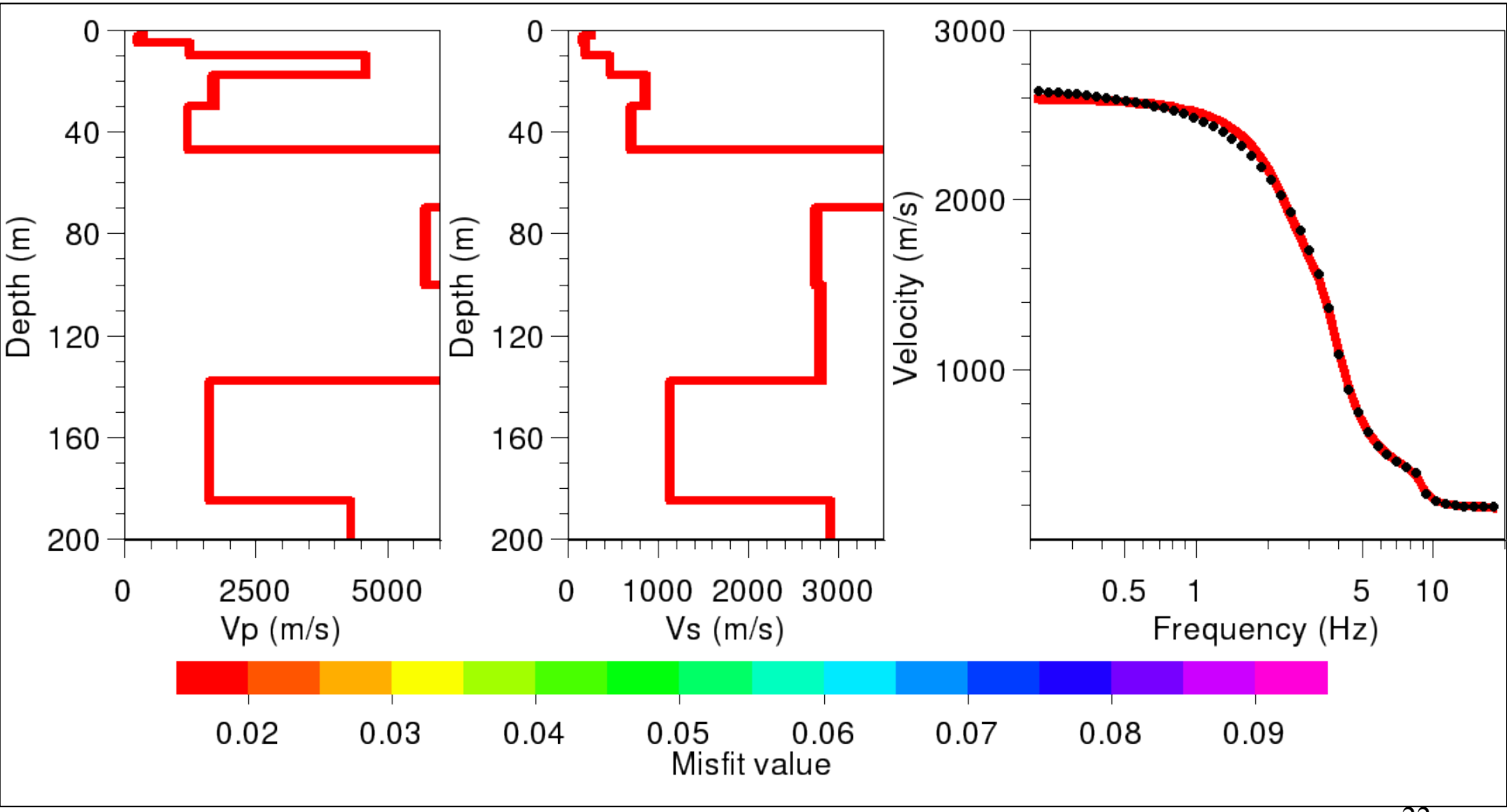


How to define parameters for this type of model?

- fixed thicknesses, Poisson's ratios fixed, free Vs in each layer (classical approach in Herrmann's codes)

- free thicknesses, free Vs, free Vp, fixed density
BUT physical limits: conditions between Vs and Vp

Dangers of classical least square approach surface wave inversion



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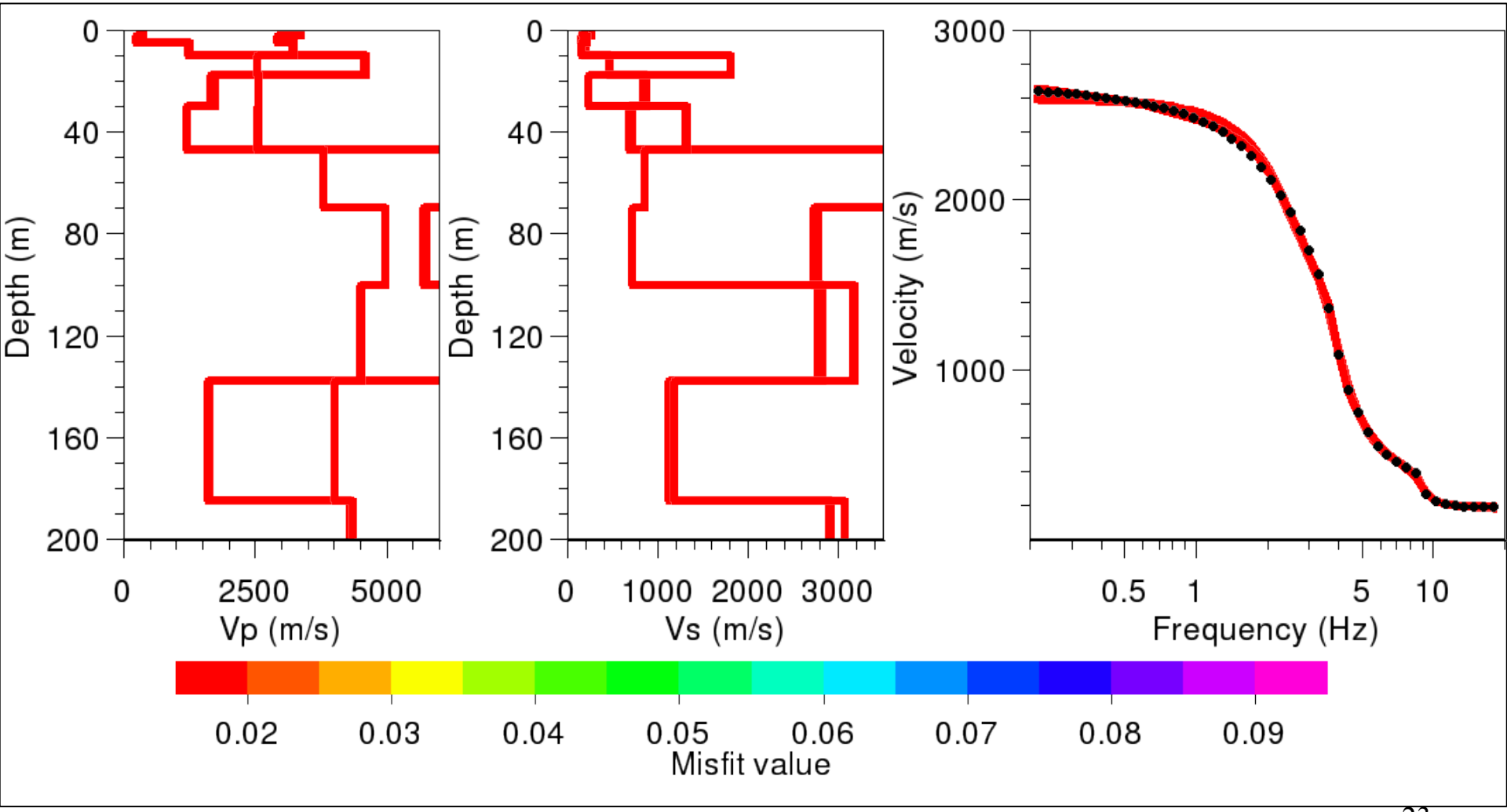

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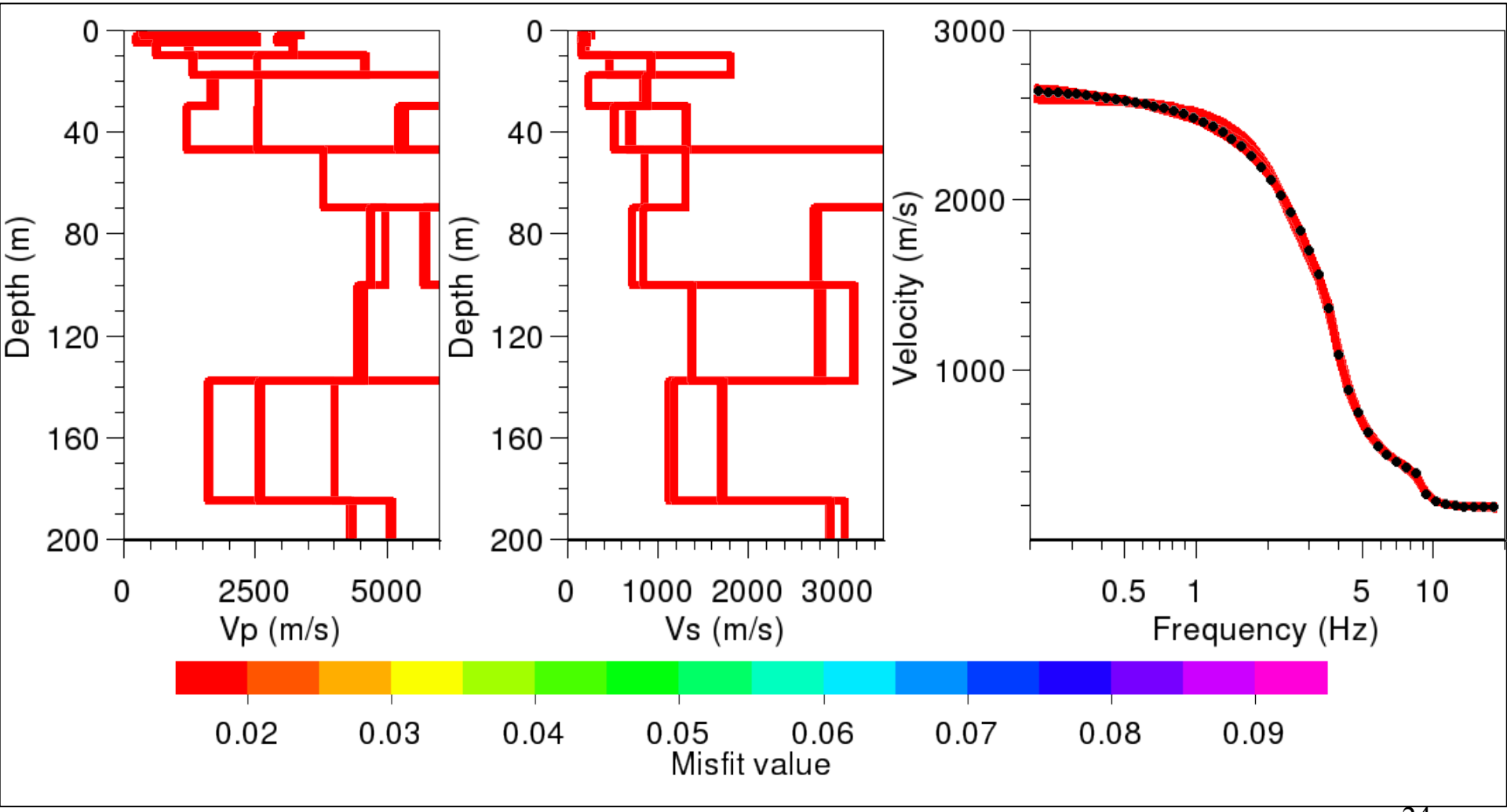

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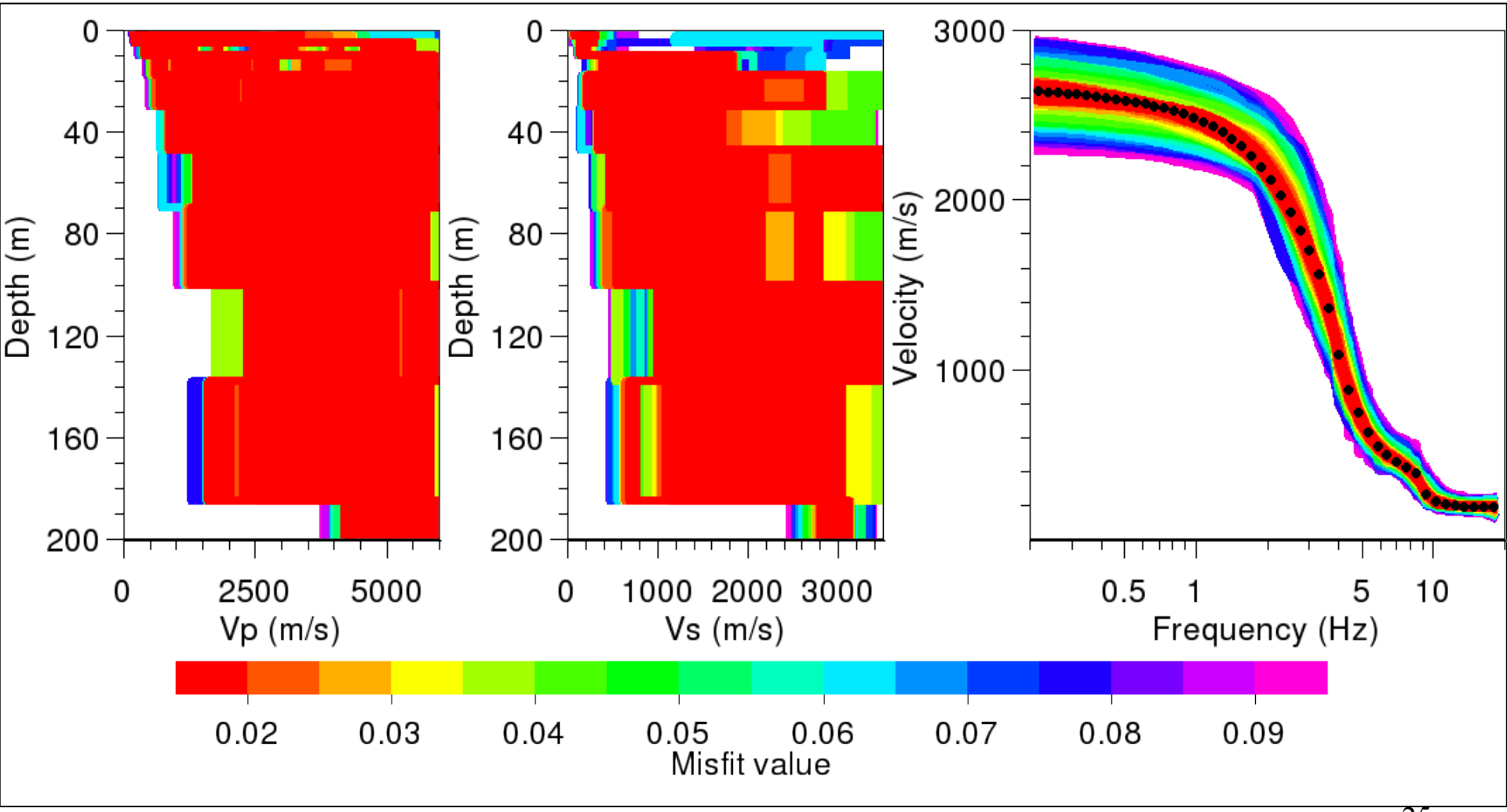
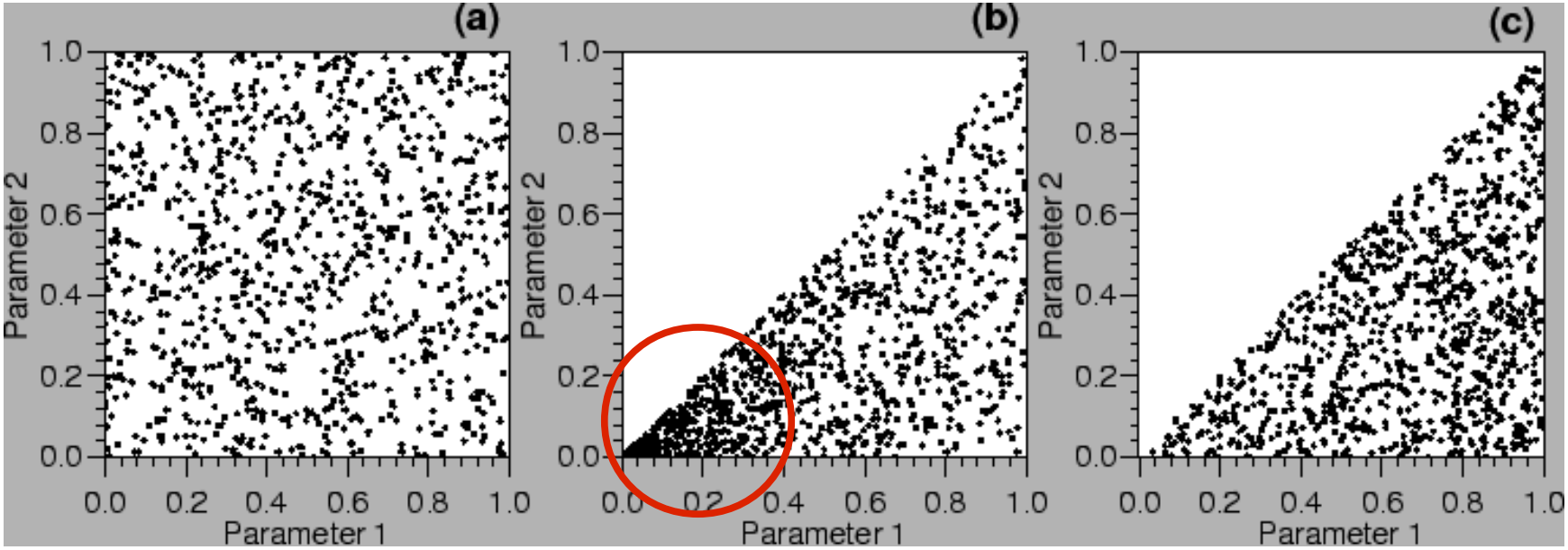


Irregular parameter space boundaries

Wathelet, M. (2008). An improved neighborhood algorithm: parameter conditions and dynamic scaling. Geophysical Research Letters, 35, doi:10.1029/2008GL033256

Sambridge: box (fixed range for all parameters)

↳ Solution to introduce conditions: variable change

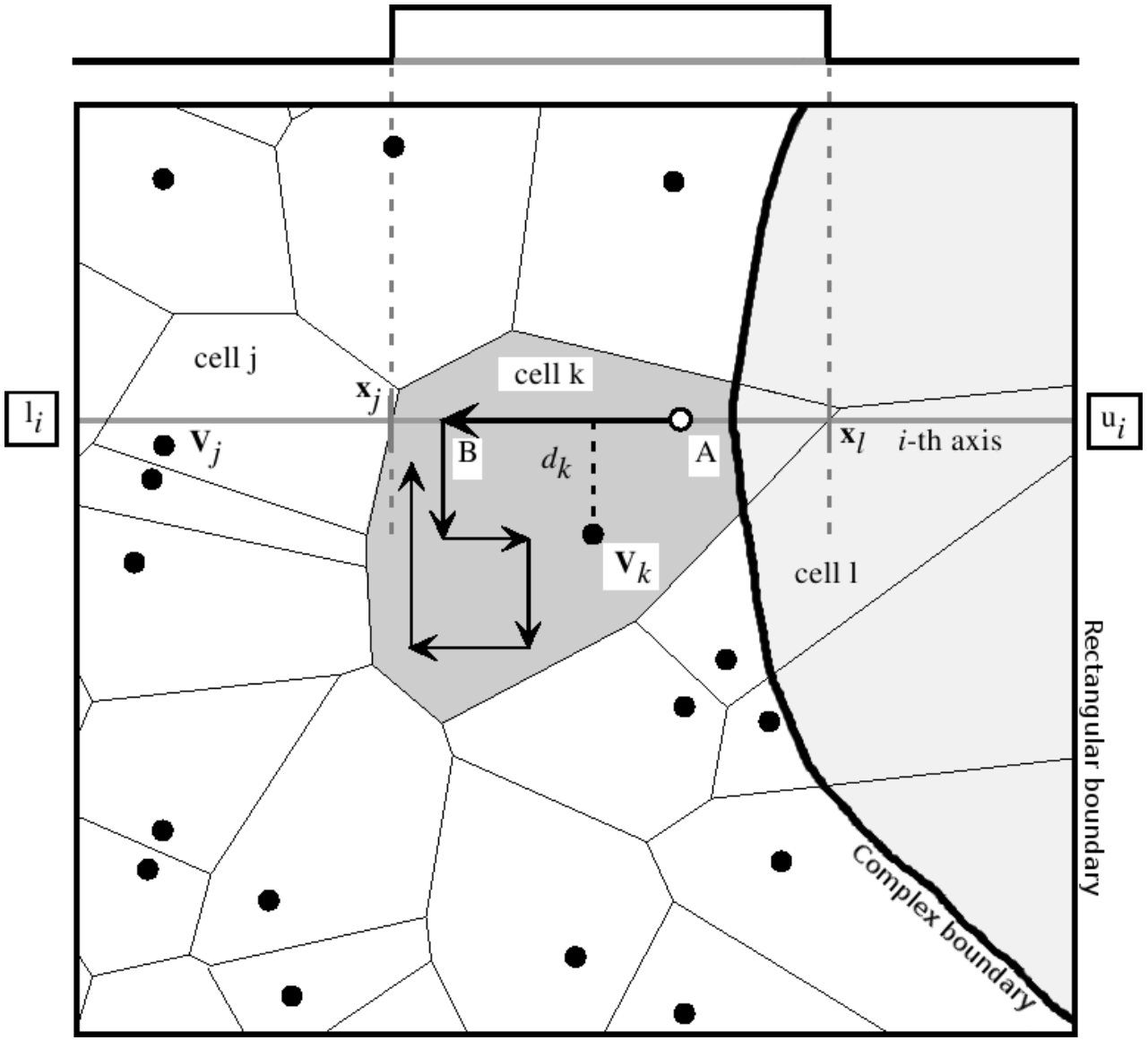


Uncontrolled Prior information

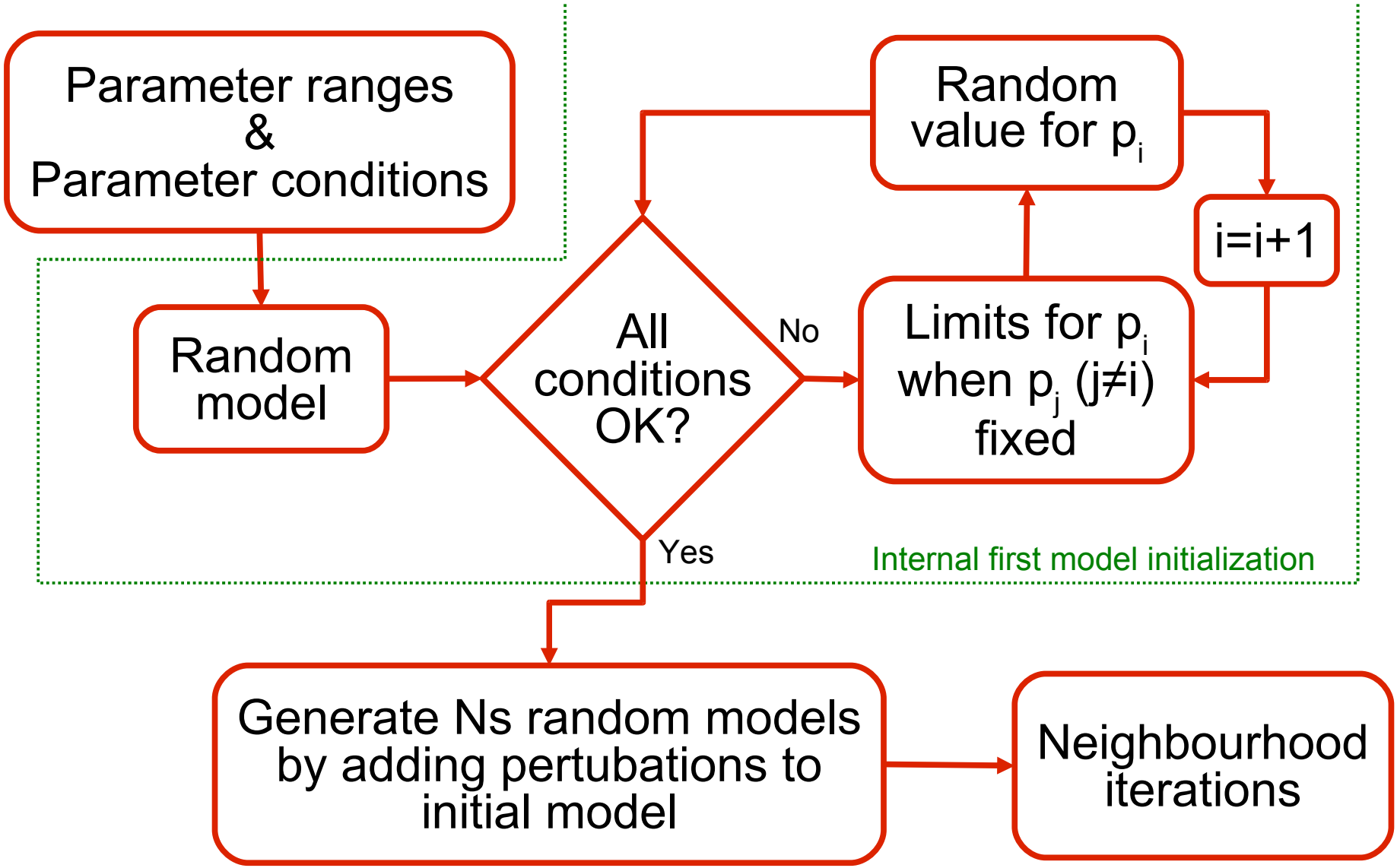
A modified Neighborhood kernel: irregular parameter boundaries

From model A add “valid” random perturbations so that model B stays in cell k

Loop over all axes



Conditions in a Neighbourhood Algorithm



Parameterization of a ground structure

- Vp and Vs as free parameters: Poisson's ratio limitations

$$\text{Poisson's ratio} = \frac{V_s^2 - \frac{V_p^2}{2}}{V_s^2 - V_p^2}$$

Usual values for soft soil & rocks
From 0.2 to 0.5

- Thickness versus depth parameters

$$depth[i] > depth[i-1]$$

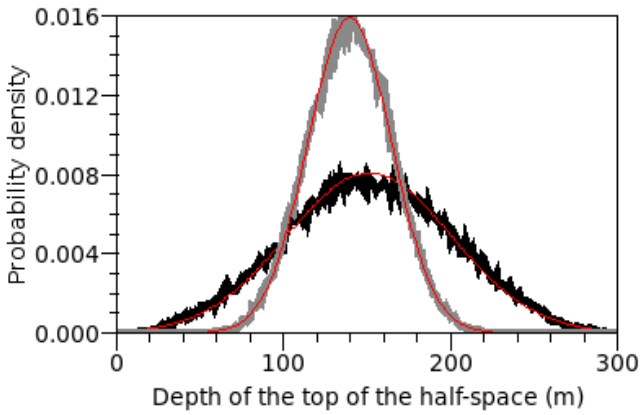
- Avoid Low Velocity Zones

$$V[i] > V[i-1]$$

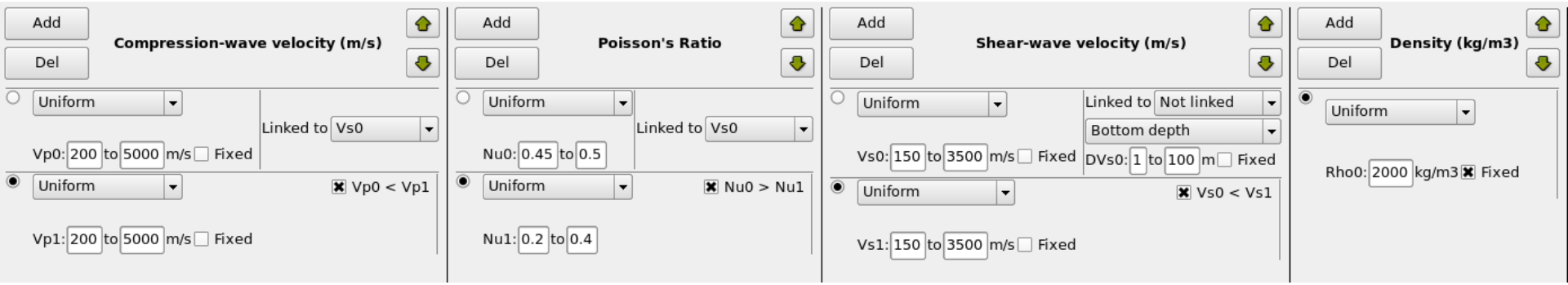
- Parameters for non-uniform layers (gradients)

Vt = Velocity at top
Vb = Velocity at bottom
Power law gradient: Vb>Vt & Vb<Vt+delta

Uncontrolled prior distribution due to a sum of parameters



Conversion between Vs-Vp ground model and a conditional parameter space



5 parameters

Rectangular limits

200 < TopVp0 < 5000 m/s

200 < TopVp1 < 5000 m/s

150 < TopVs0 < 3500 m/s

1 < DVs0 < 100 m

150 < TopVs1 < 3500 m/s

TopRho0=2000 kg/m3

Special limits

Poisson's ratio

TopVp1 > TopVp0

TopVs1 > TopVs0

Feature summary

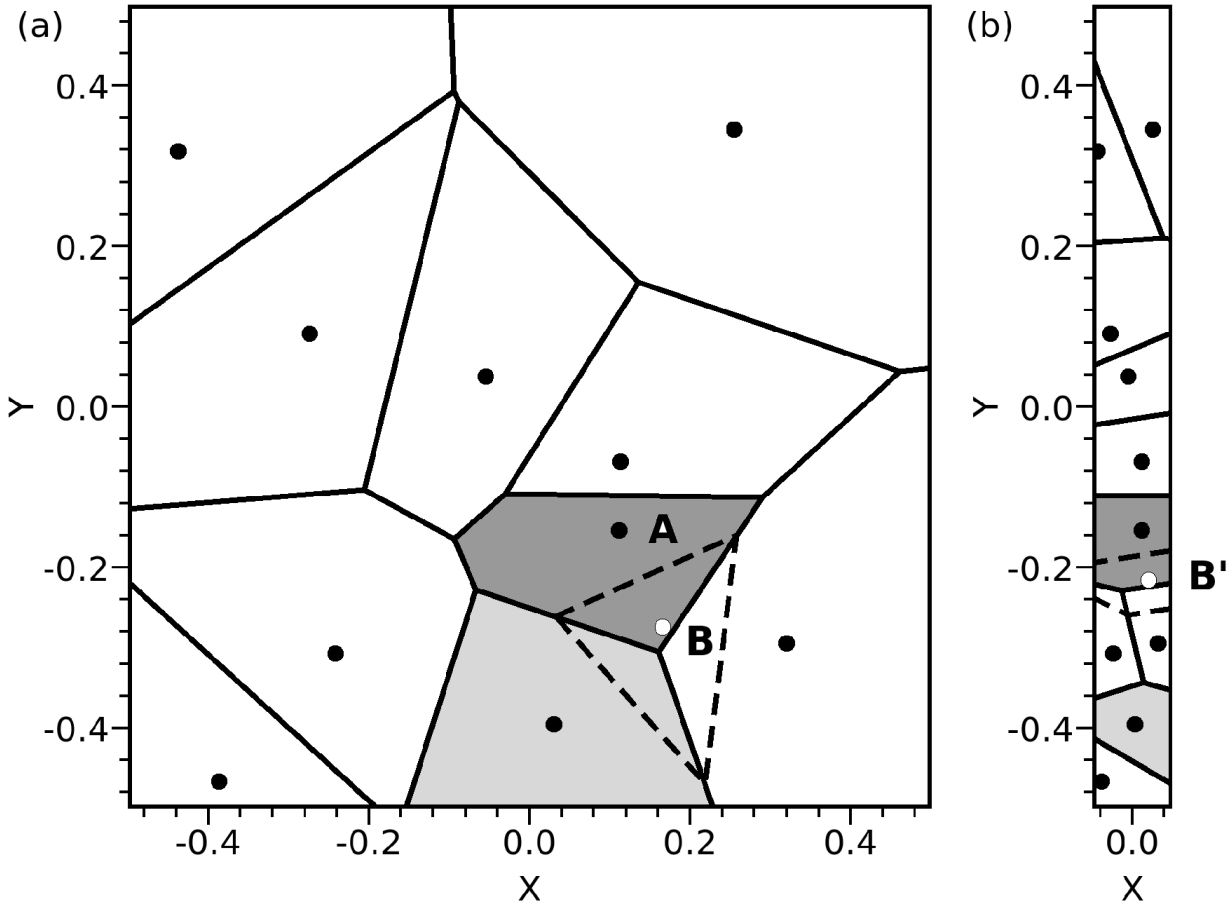
- Uniform or gradient layers (power law or linear)
- Fixed parameter range for prior information
- Uncorrelated Vp, Vs and density profiles
- Depth and/or thickness
- Full control over Low Velocity Zones
- Custom conditions (impedance contrast)
- Fine Poisson's ratio limits

5. Other recent features

Dynamic parameter scaling

An interesting property of Voronoi cells:

Effect of axis scaling

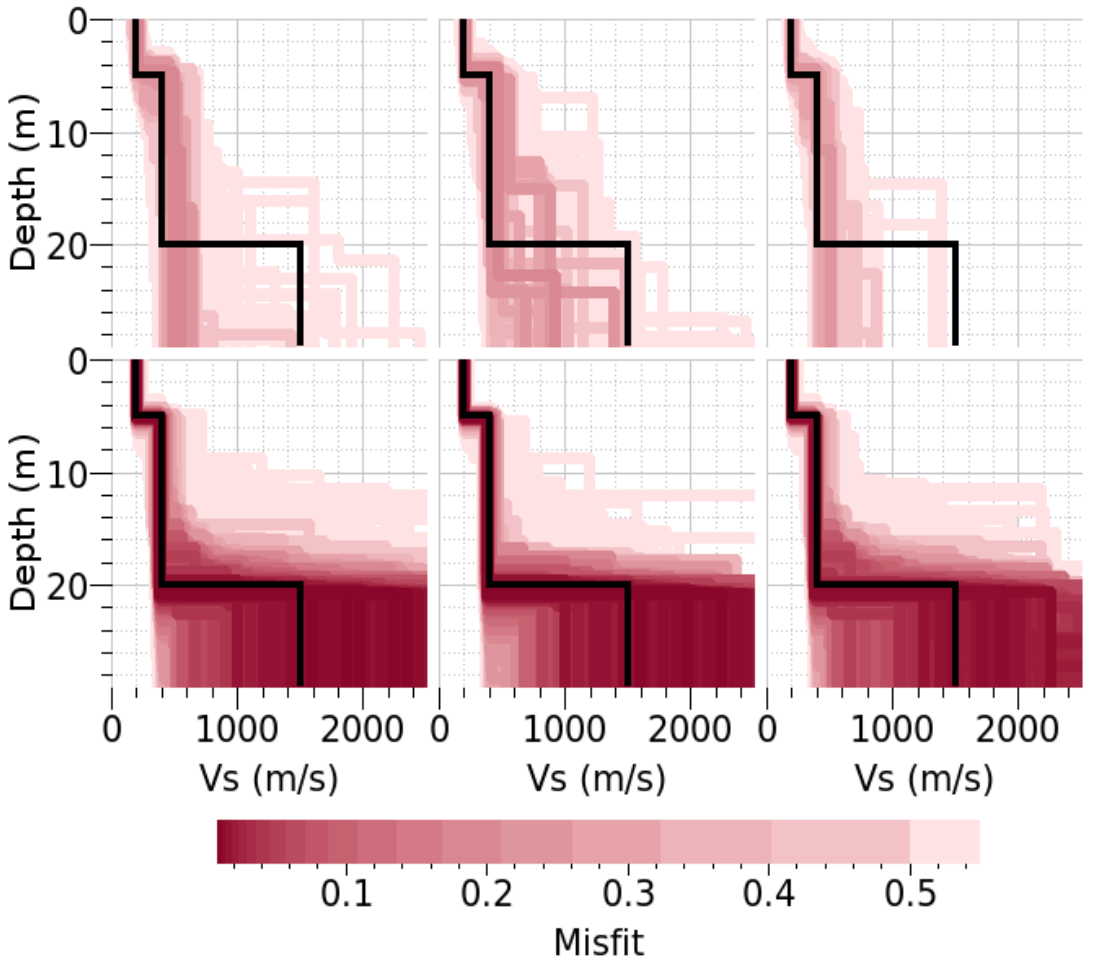


NA explores always best along the smallest axis range

Boosting exploration capabilities

Static scaling

Dynamic scaling

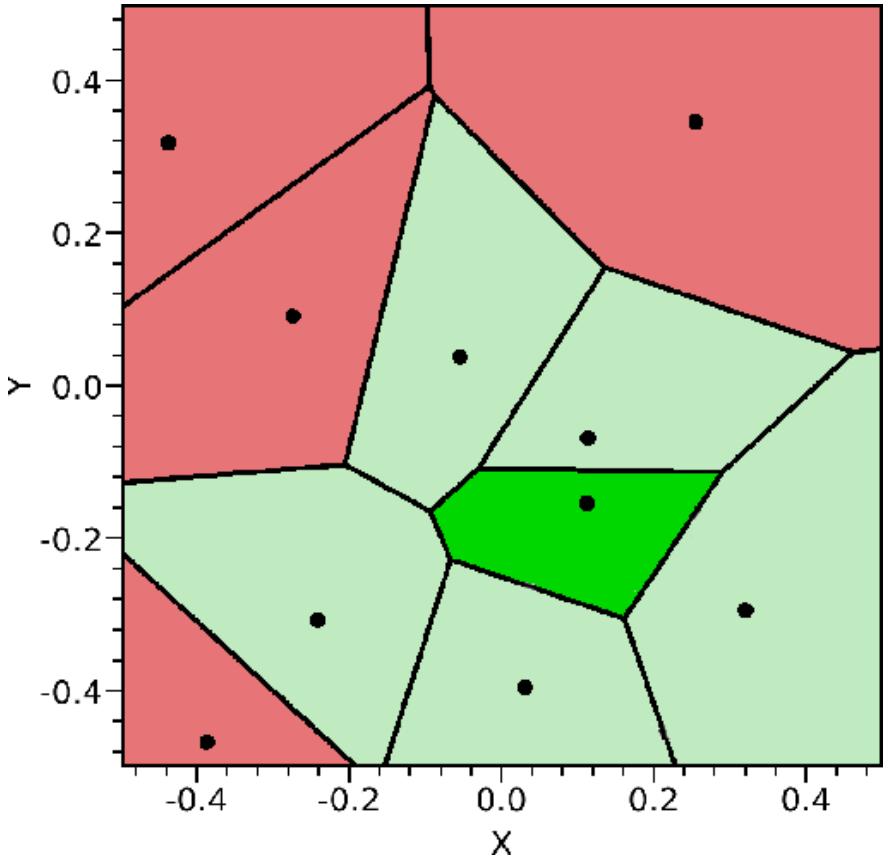


Various random seeds: robustness

5. Other recent features
Active models

When number of iteration becomes high (e.g. itmax>200), a classical NA slows down in a dramatic way

Restrict the exploration to "useful" or "active" cells only



5. Other recent features
Discrete NA

- Avoid precision errors when running many iterations
- Control of the precision of each parameter
- Linear or log scales

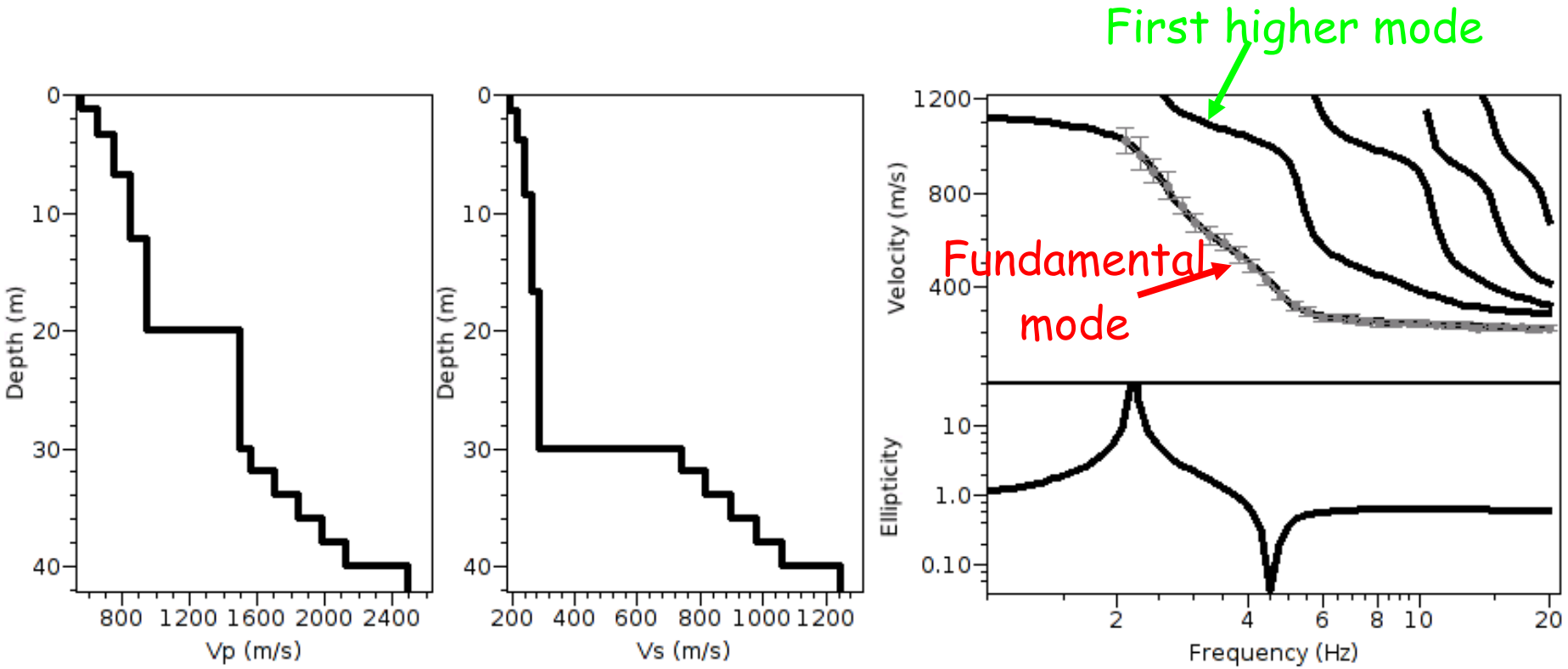
LINEAR: absolute precision

LOG: relative precision

For surface wave inversion all parameters are on a log scale with a maximum precision of 1%

6. Inversion examples

Virtual test site: Vp and Vs structure

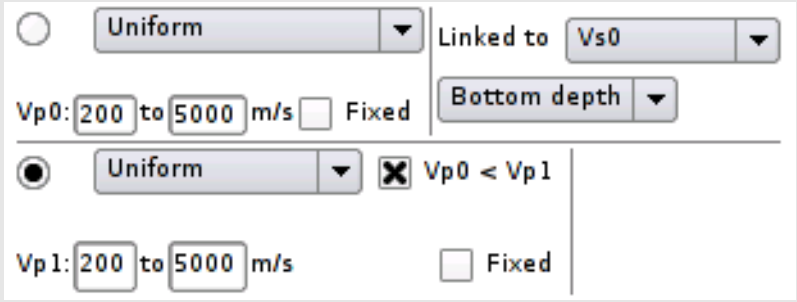


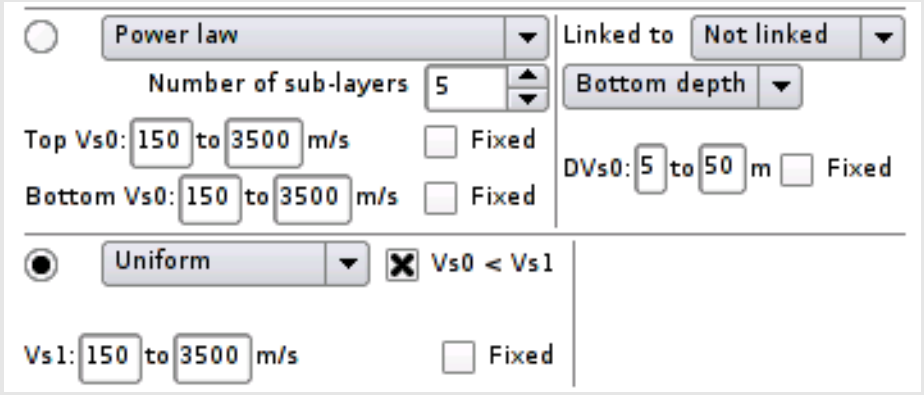
Parameterization of a 2-layer model

Vp

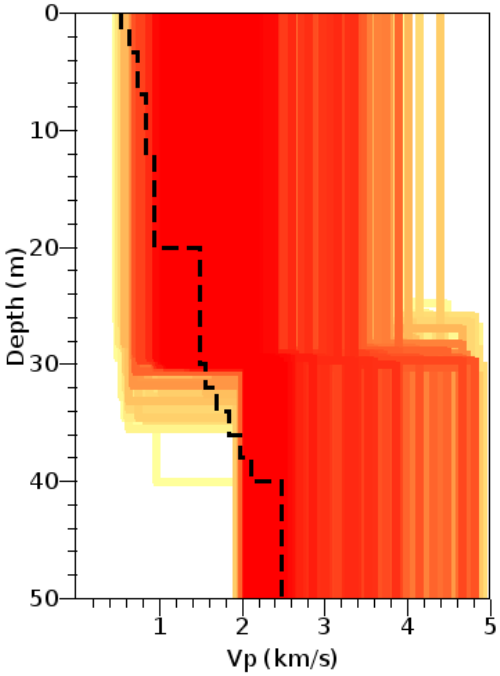
Vs

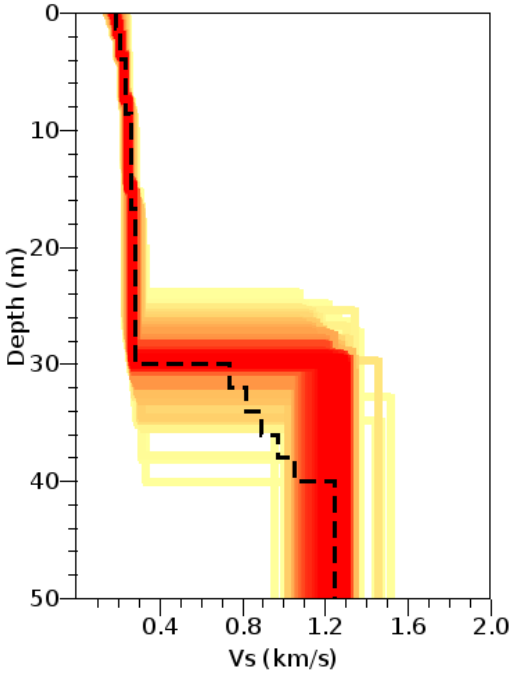
Density

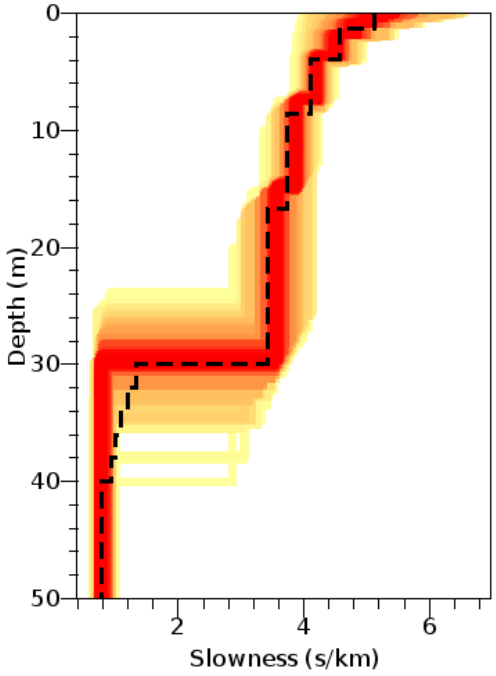


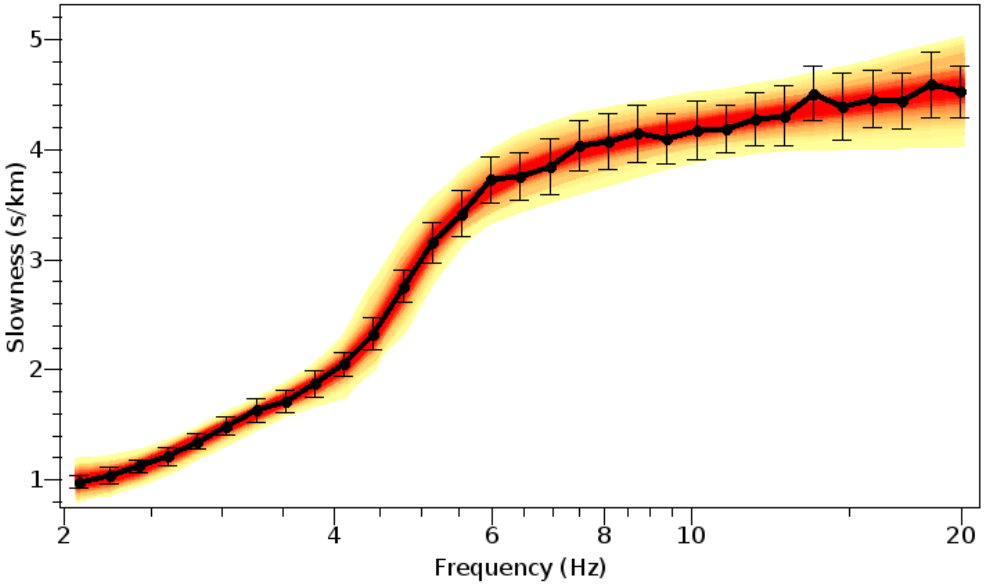


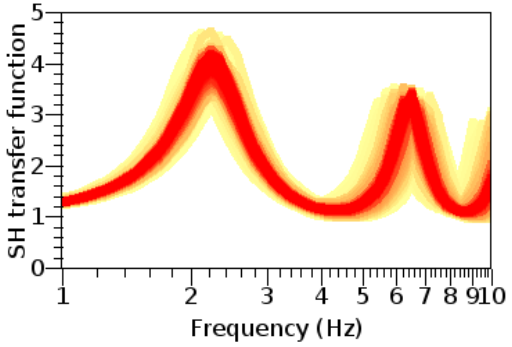
2 layers

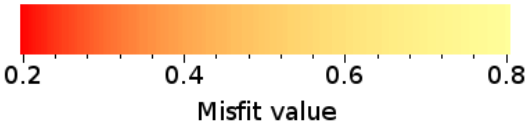

Depth (m)
Vp (km/s)


Depth (m)
Vs (km/s)


Depth (m)
Slowness (s/km)


Slowness (s/km)
Frequency (Hz)


SH transfer function
Frequency (Hz)

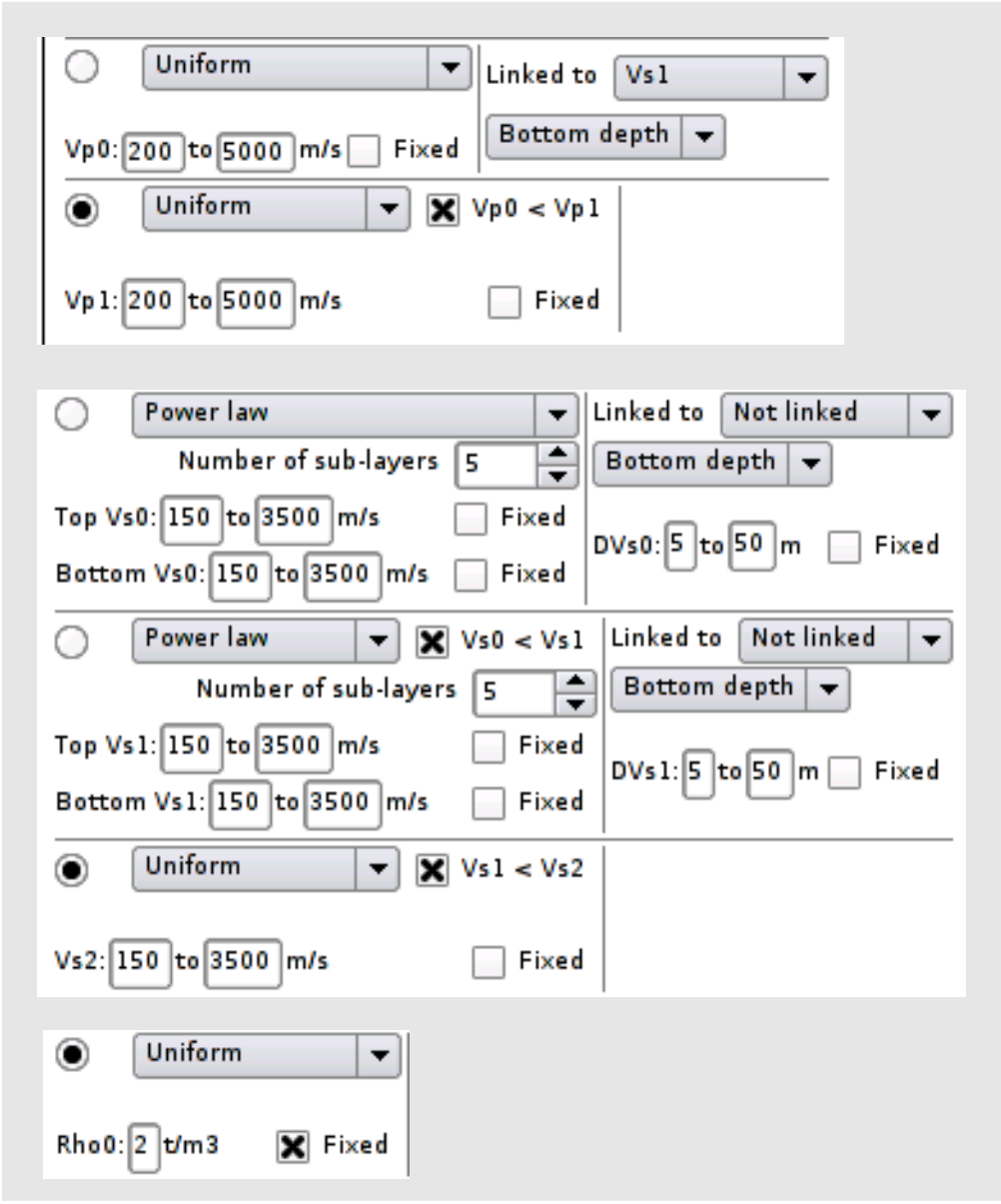

Misfit value

Parameterization of a 3-layer model

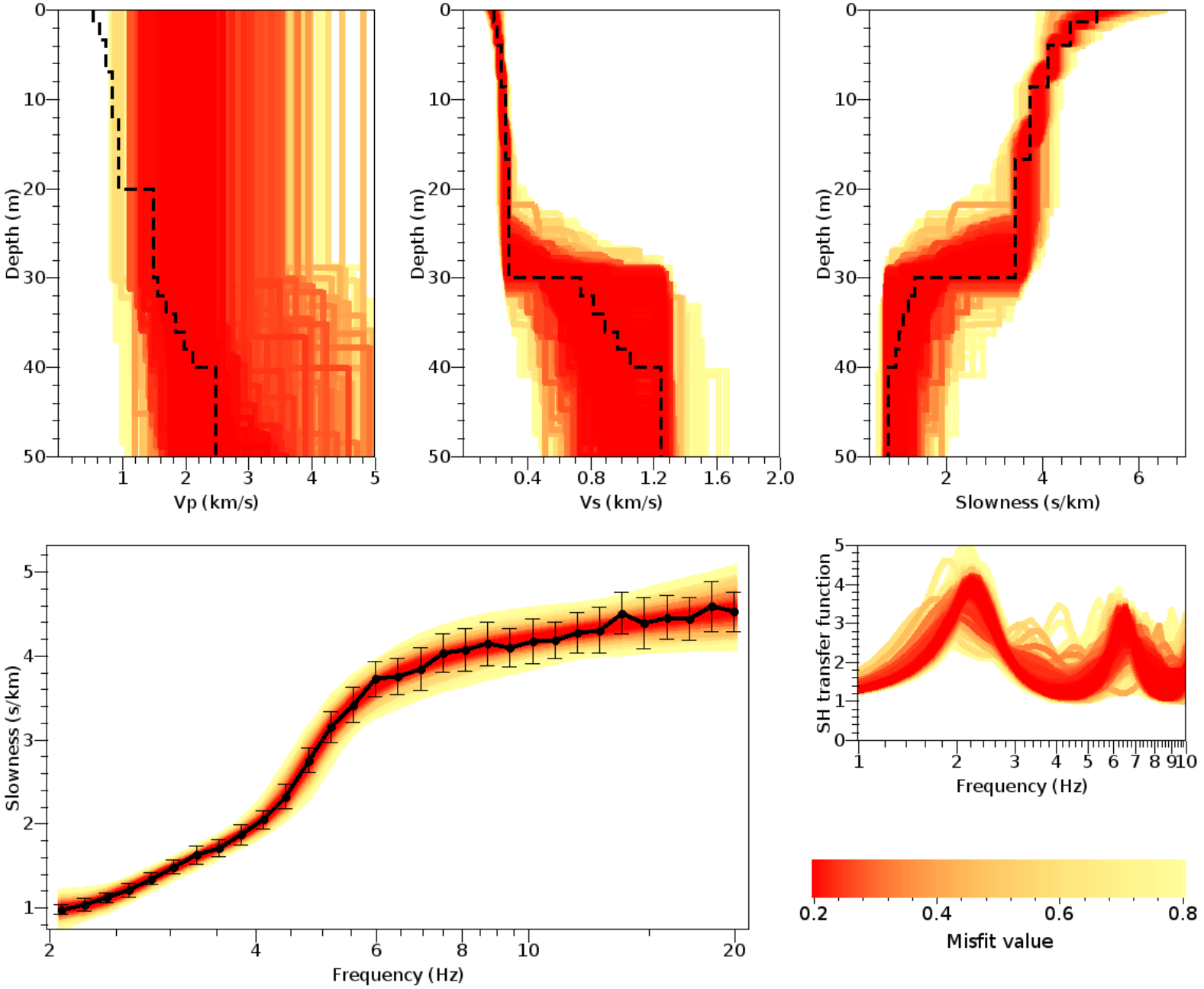
Vp

Vs

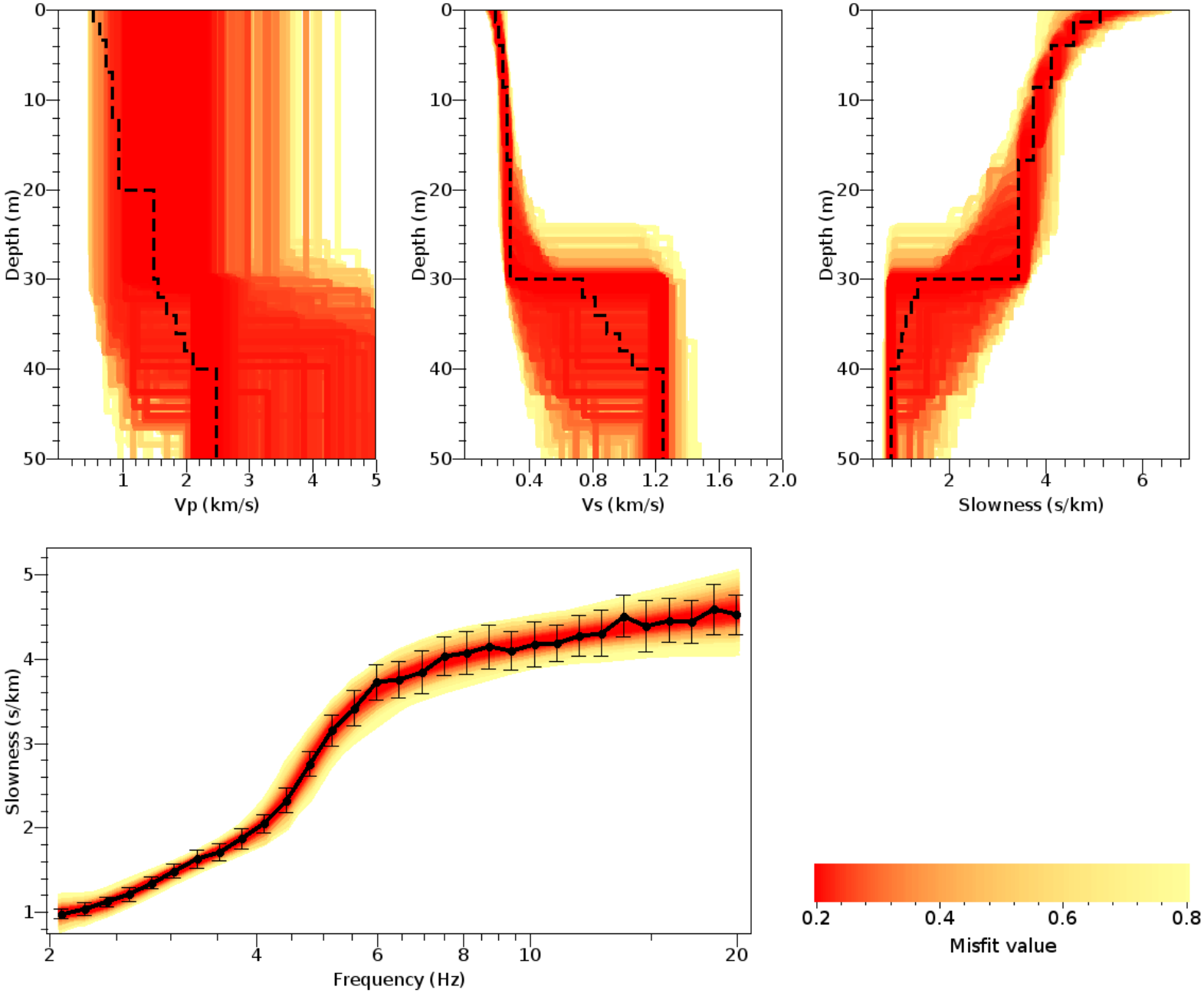
Density



3 layers


Depth (m)
Vp (km/s)
Vs (km/s)
Slowness (s/km)
Slowness (s/km)
Frequency (Hz)
SH transfer function
Frequency (Hz)
Misfit value
0.2
0.4
0.6
0.8

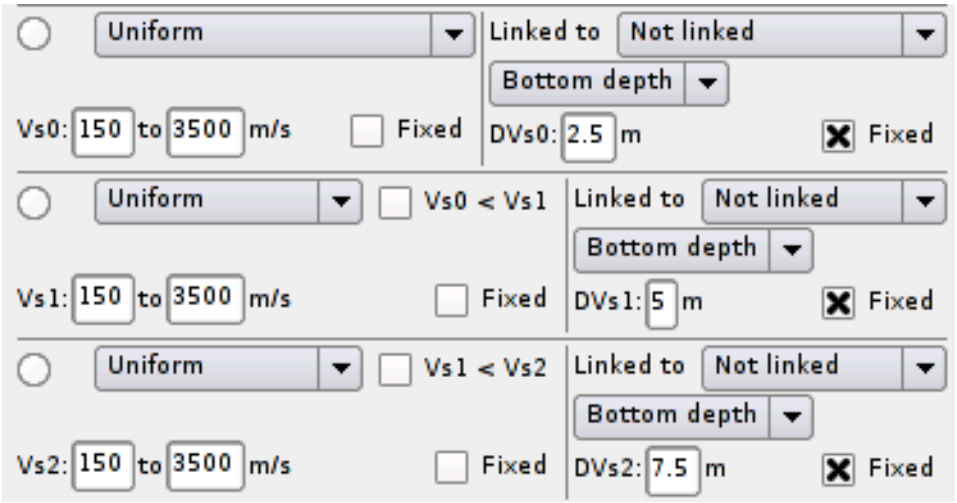
3 layers


Depth (m)
Vp (km/s)
Vs (km/s)
Slowness (s/km)
Slowness (s/km)
Frequency (Hz)
Misfit value
0.2
0.4
0.6
0.8

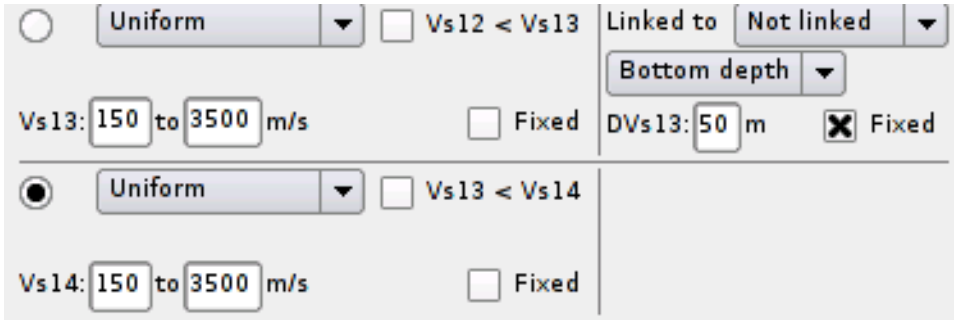
Parameterization of a 15-layer model

=> Identical to the classical approach
(Herrmann, linerization, gradient methods)

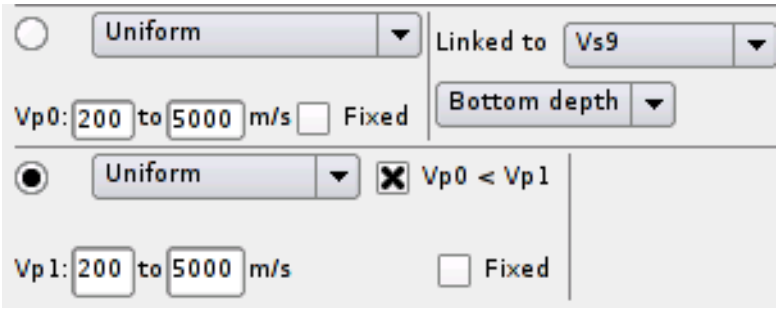
Vs



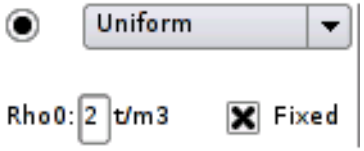
. . .



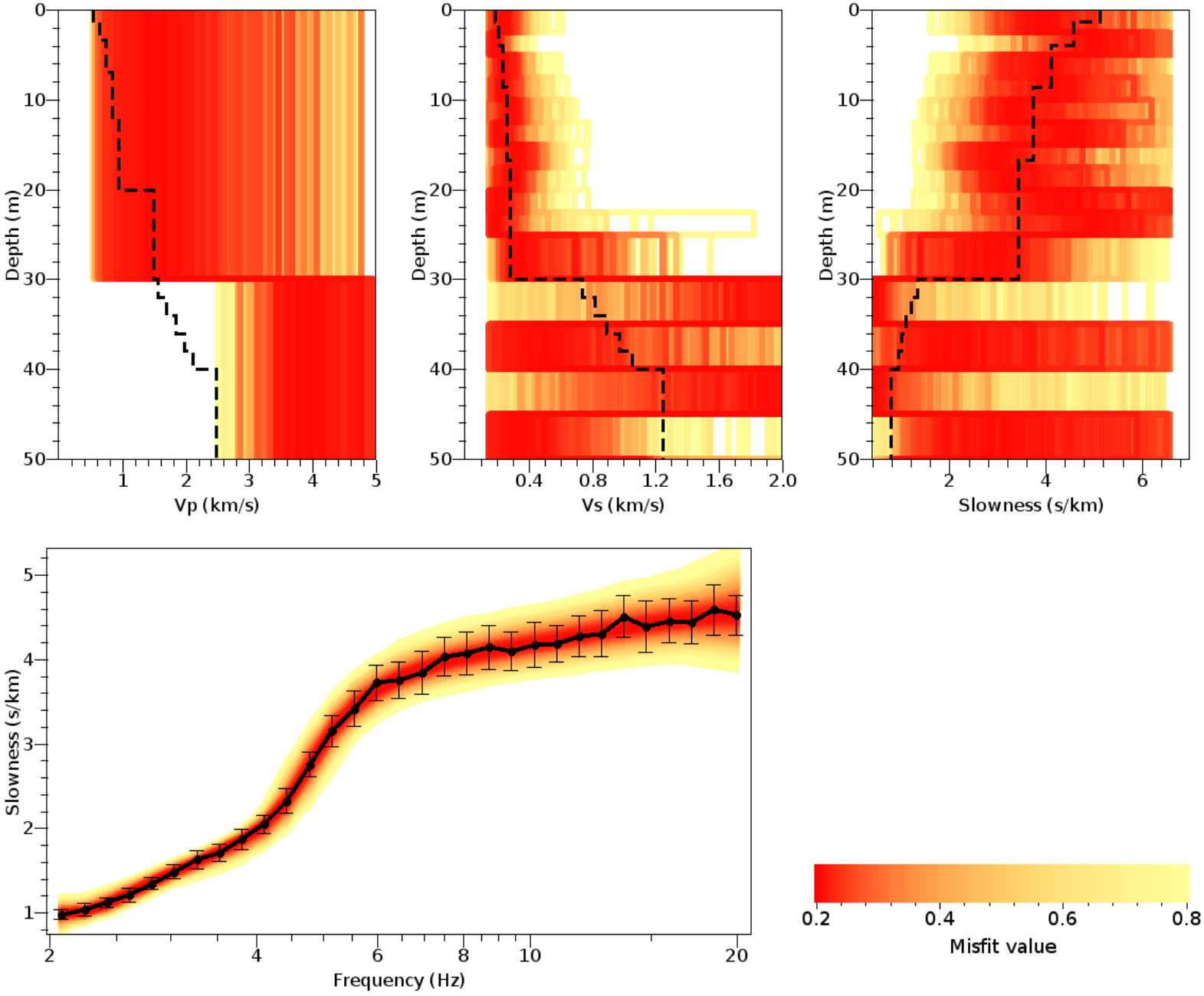
Vp



Density



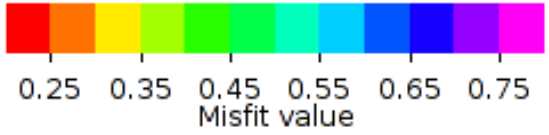
15 layers
LVZ allowed

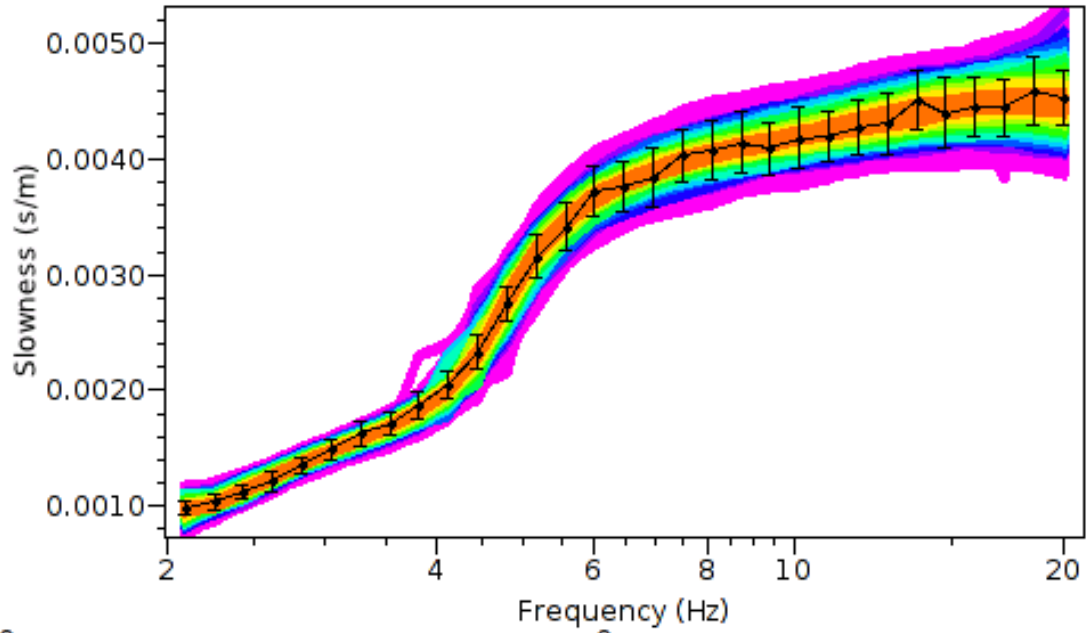


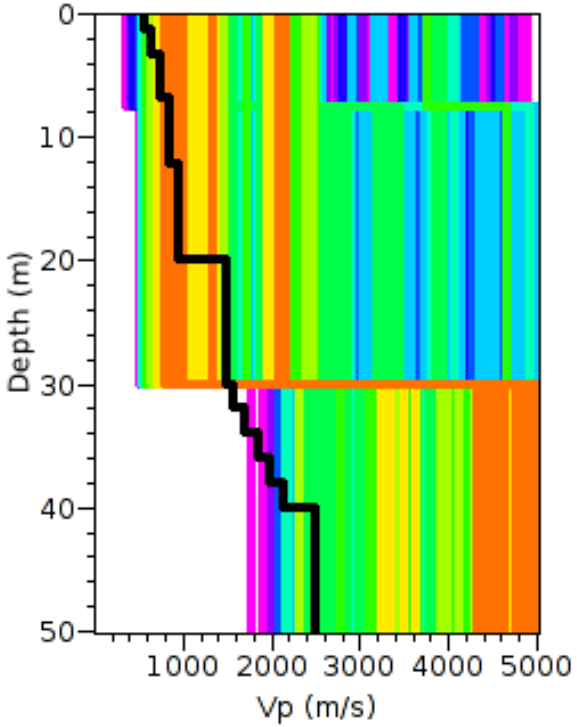
February 21st-28th 2010, Thessaloniki, Greece

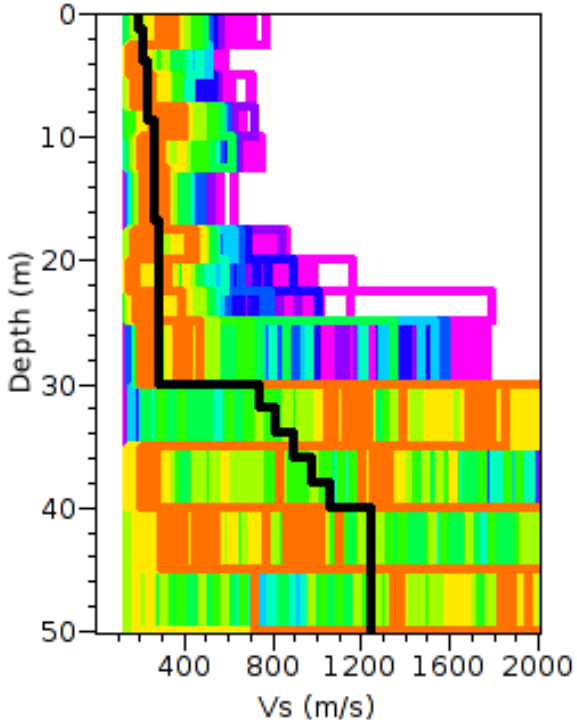
15 layers
LVZ allowed

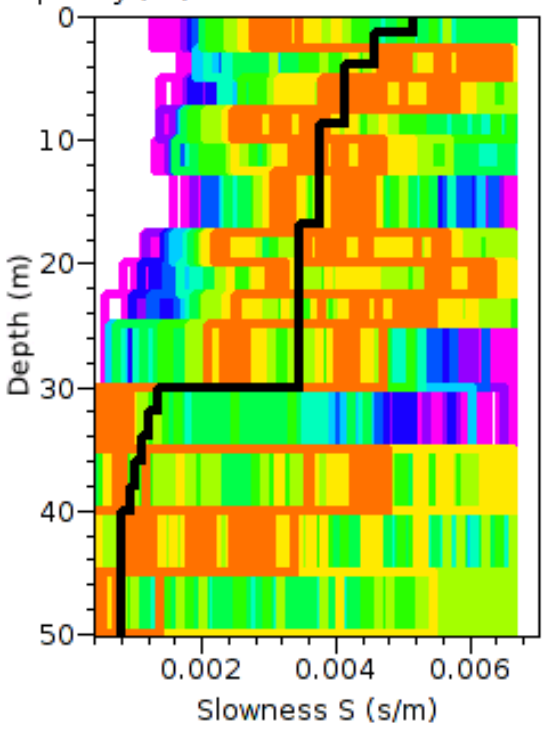
4 years ago...
22 distinct seeds
400000 models







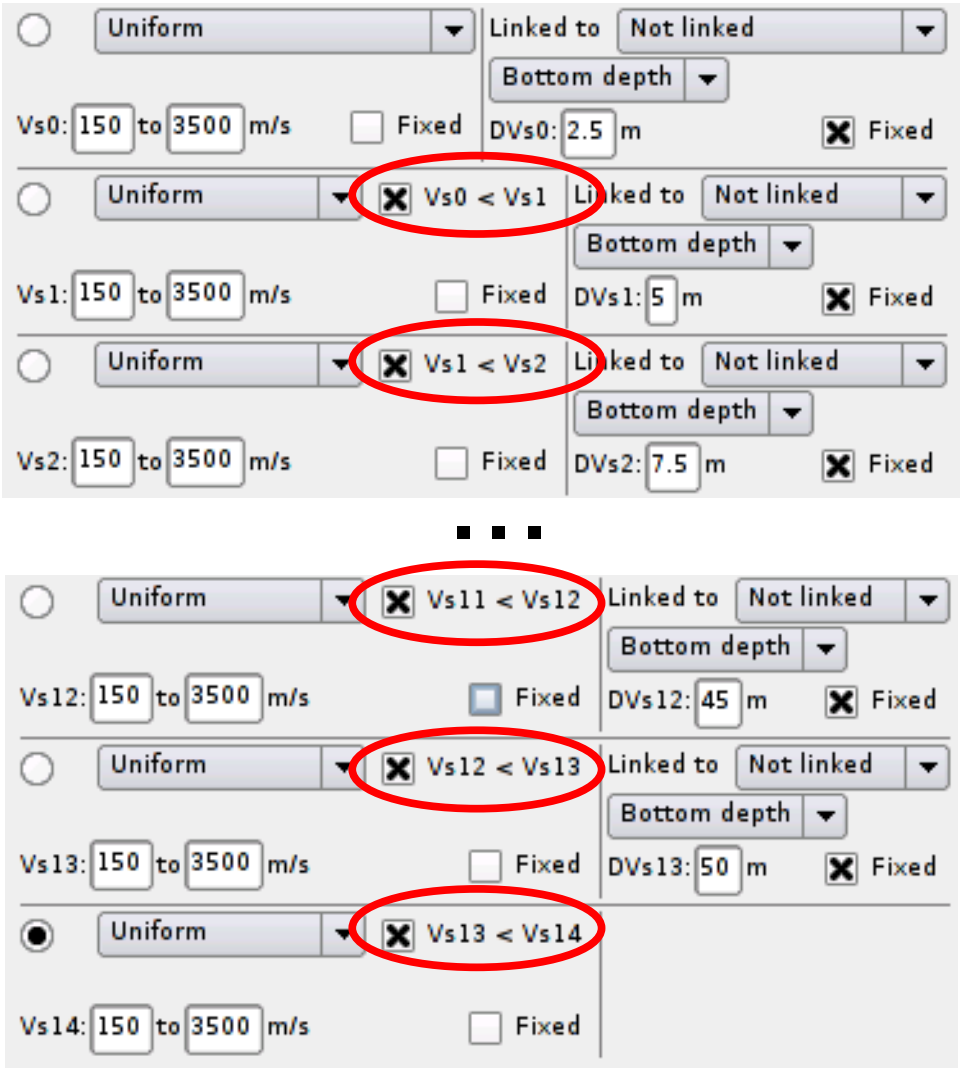




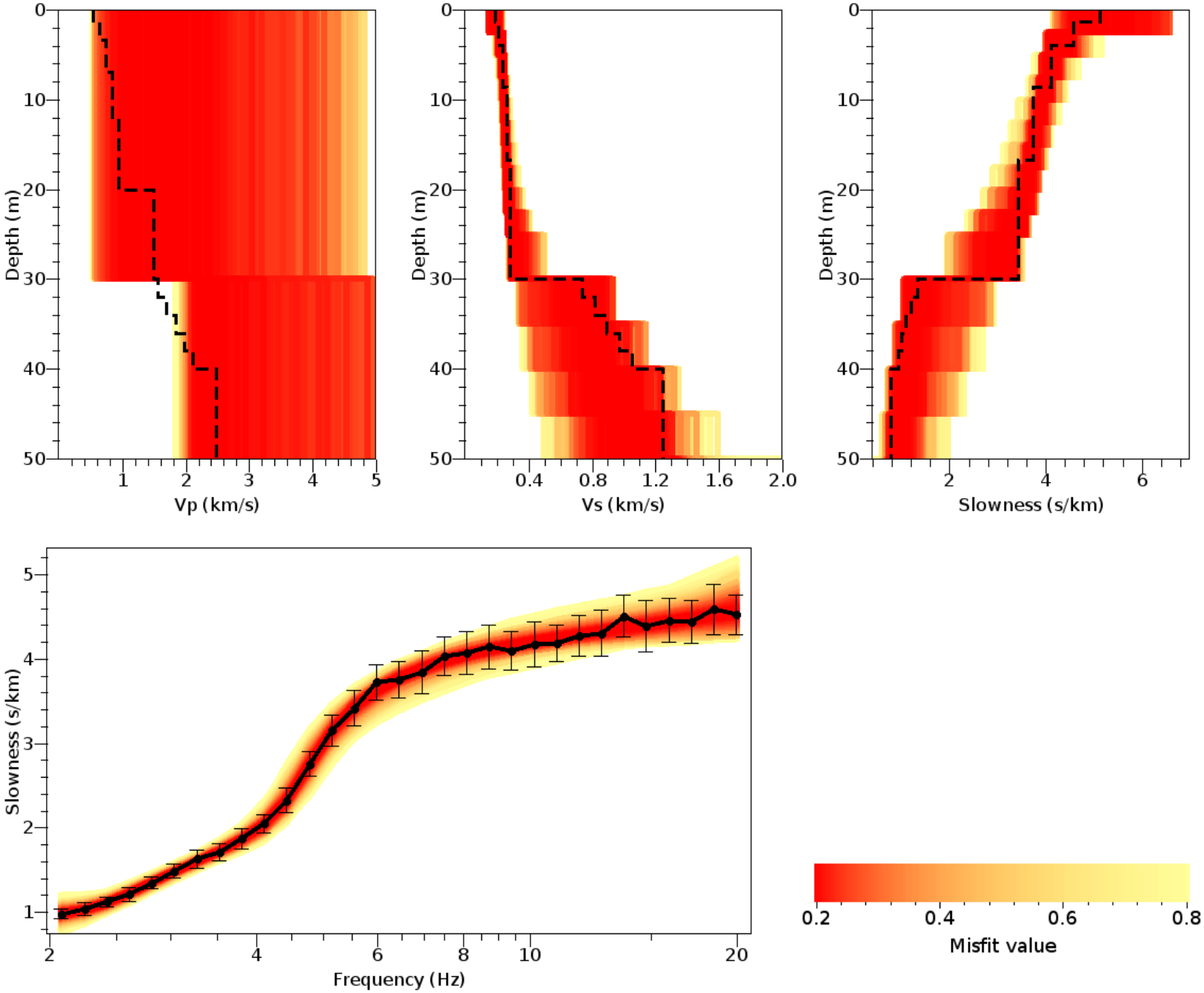
Parameterization of a 15-layer model

Controlling the presence of low velocity zones

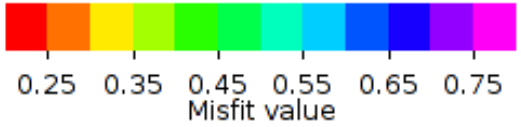
Vs

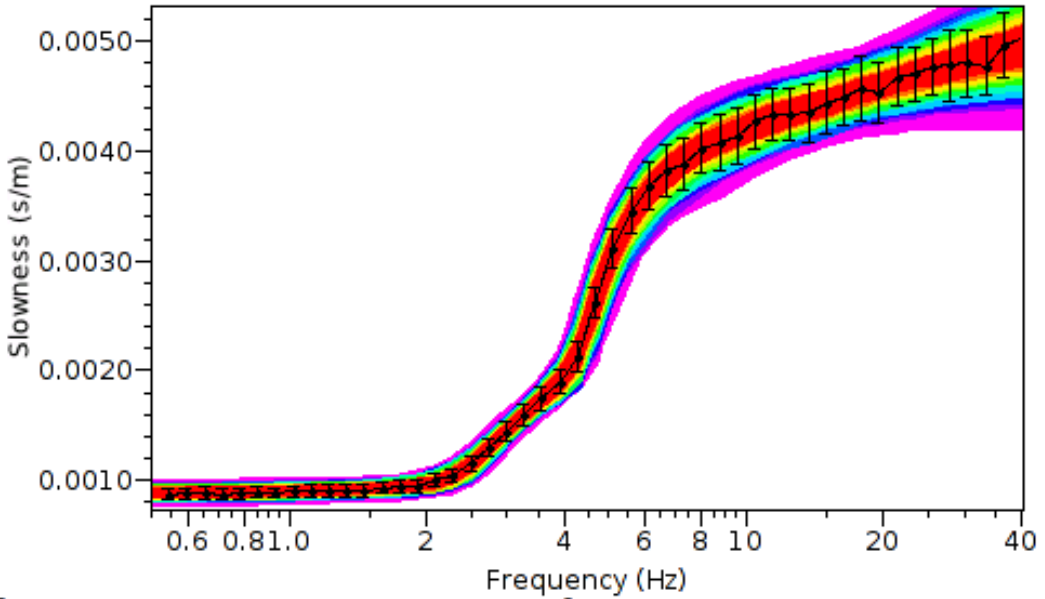


15 layers
No lvz

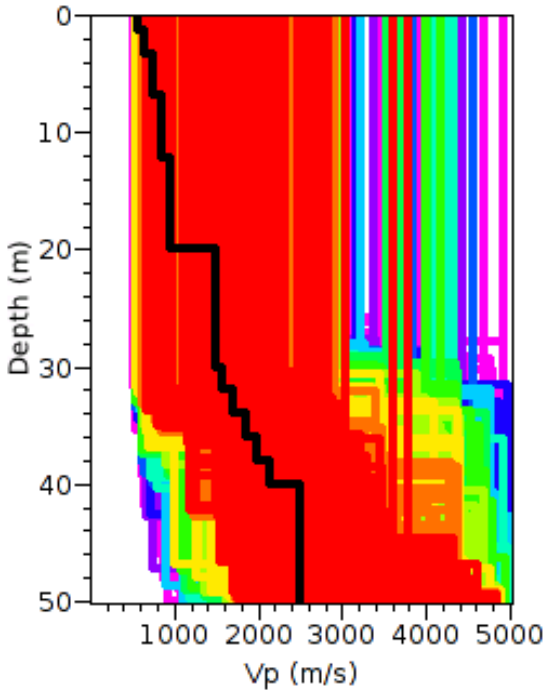


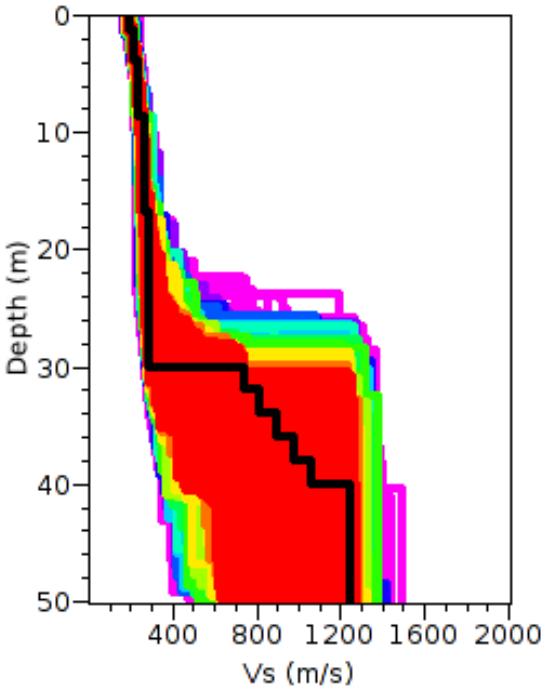
Full range dispersion curve

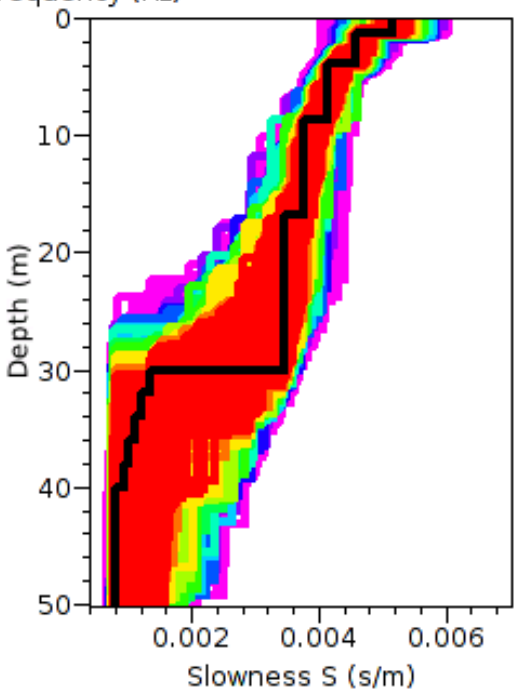




Effects of frequency range

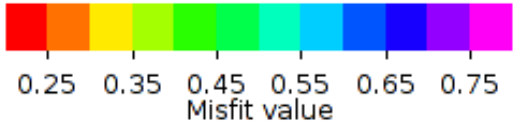


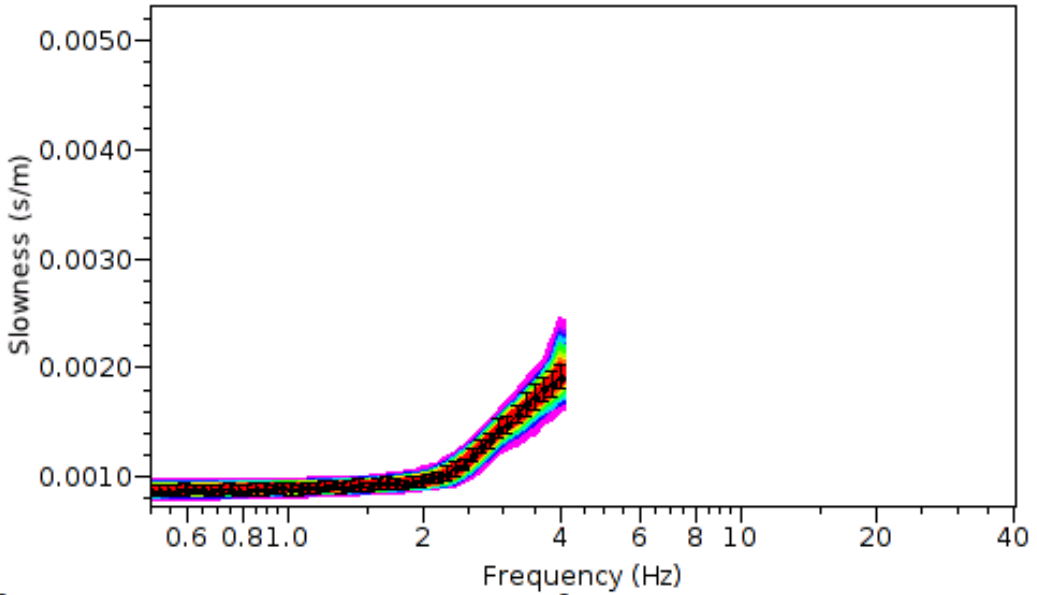


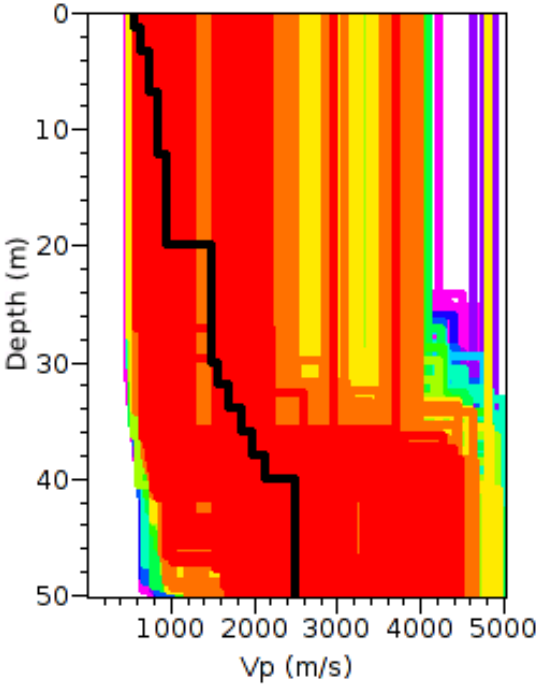


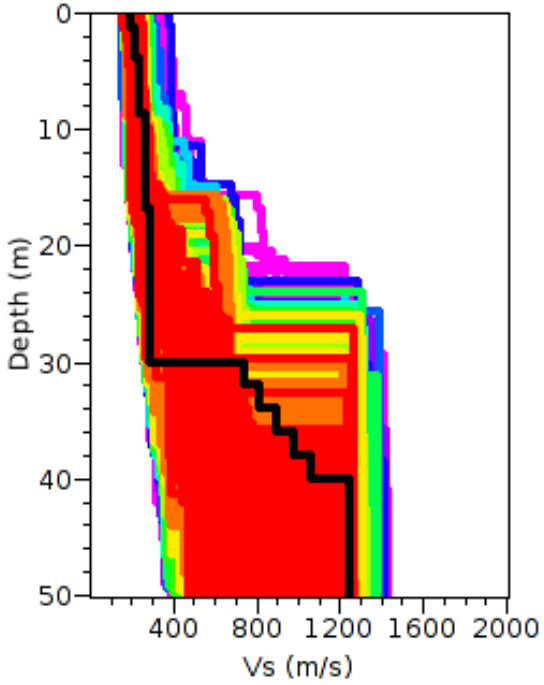
Effects of frequency range

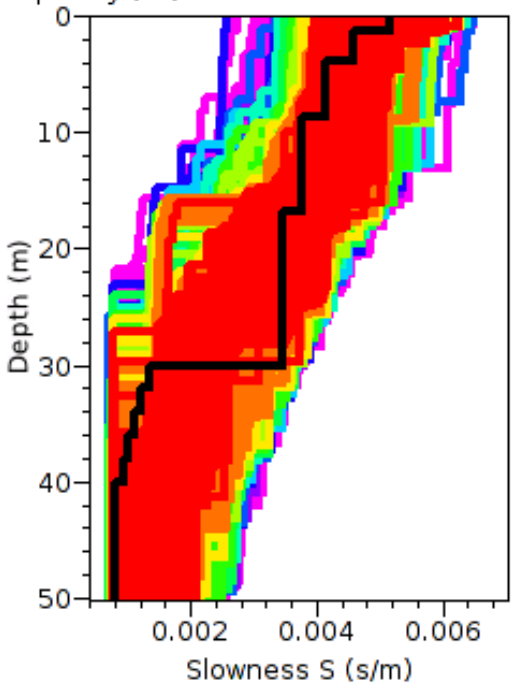
Low frequency



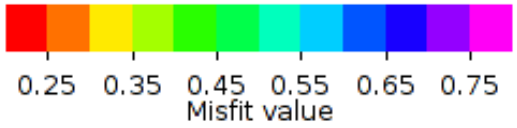


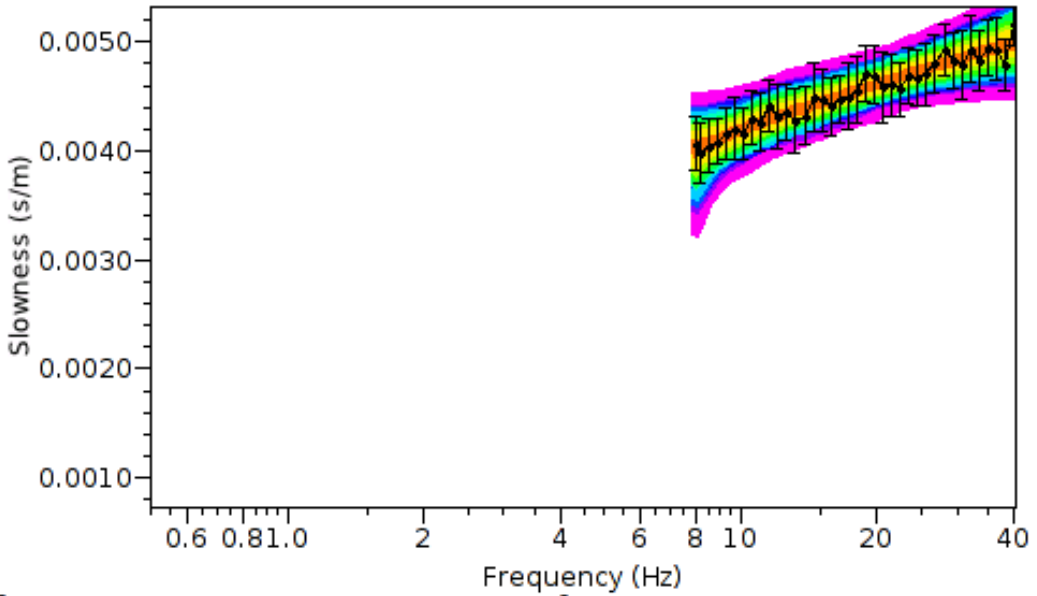


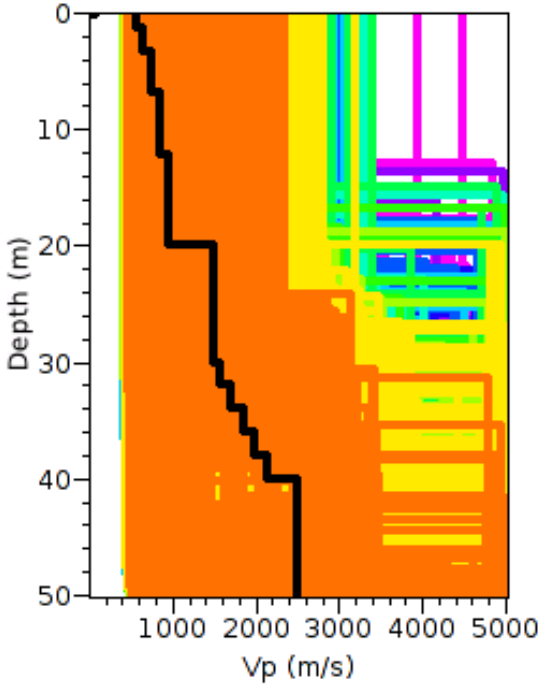


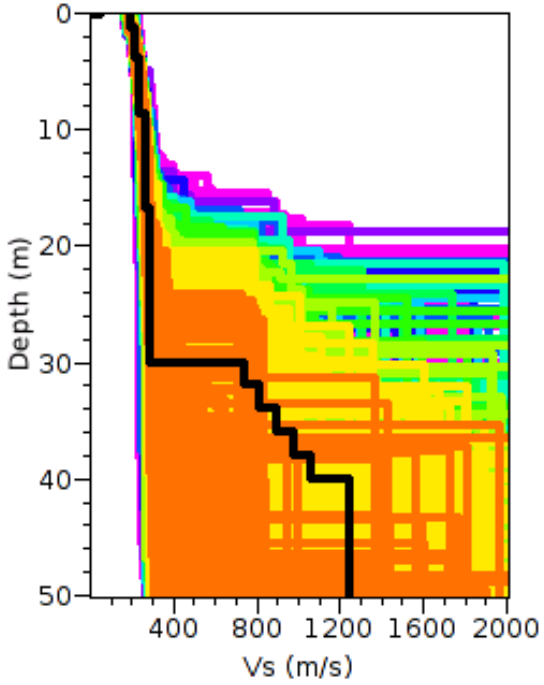


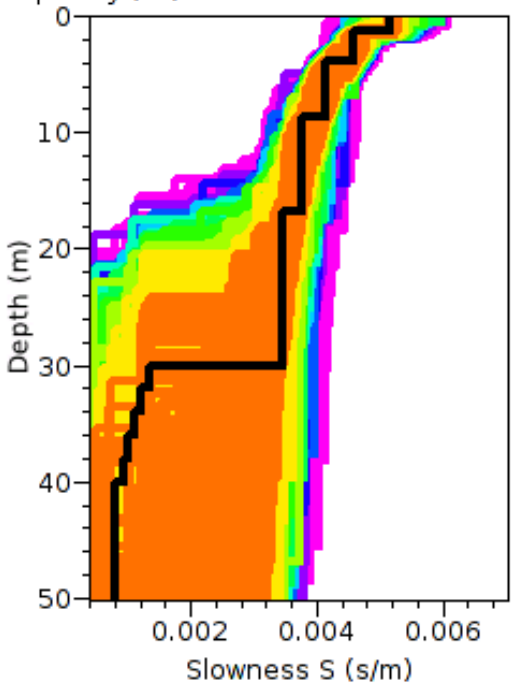
High frequency





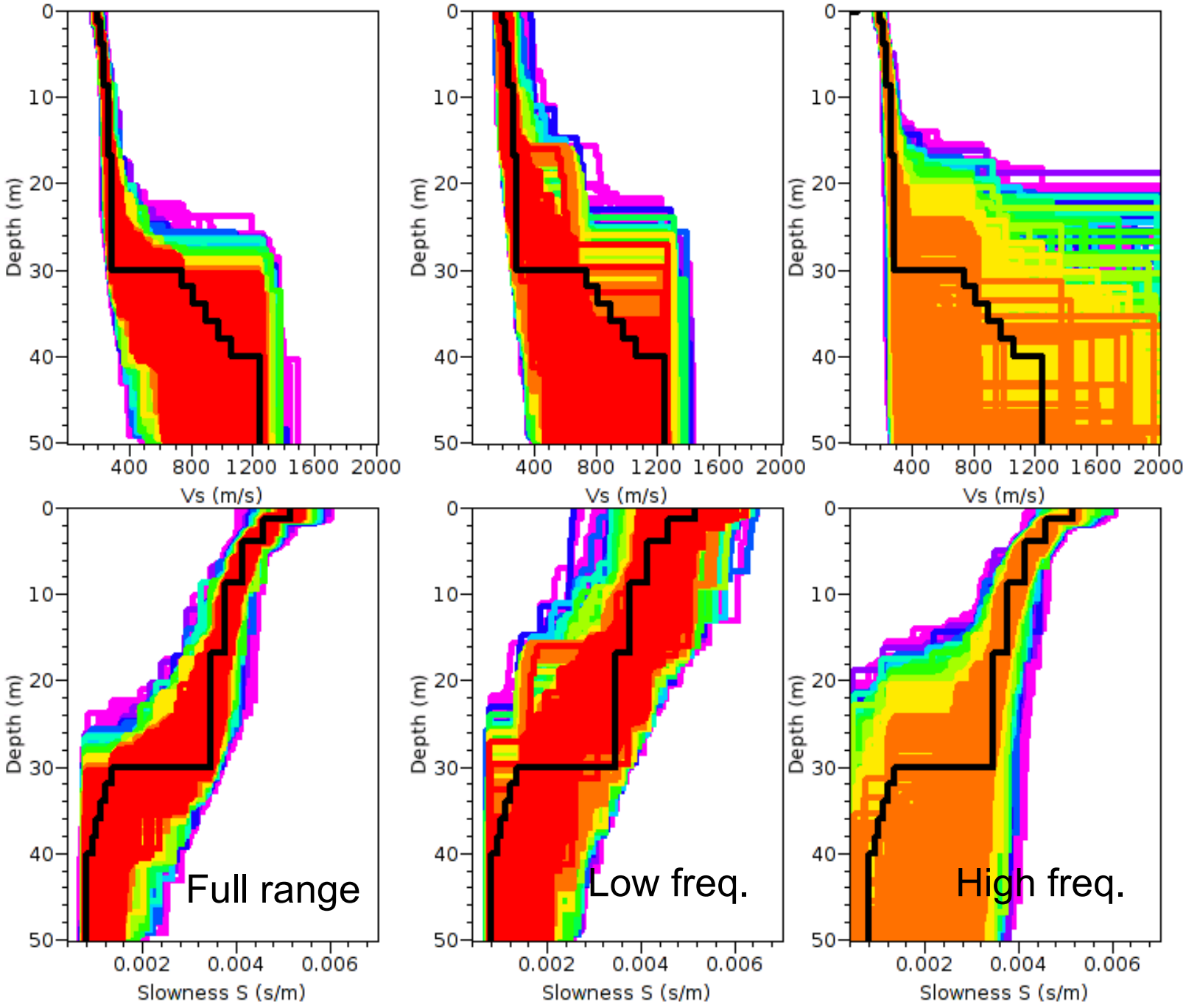




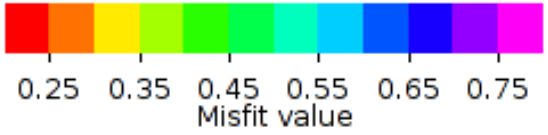


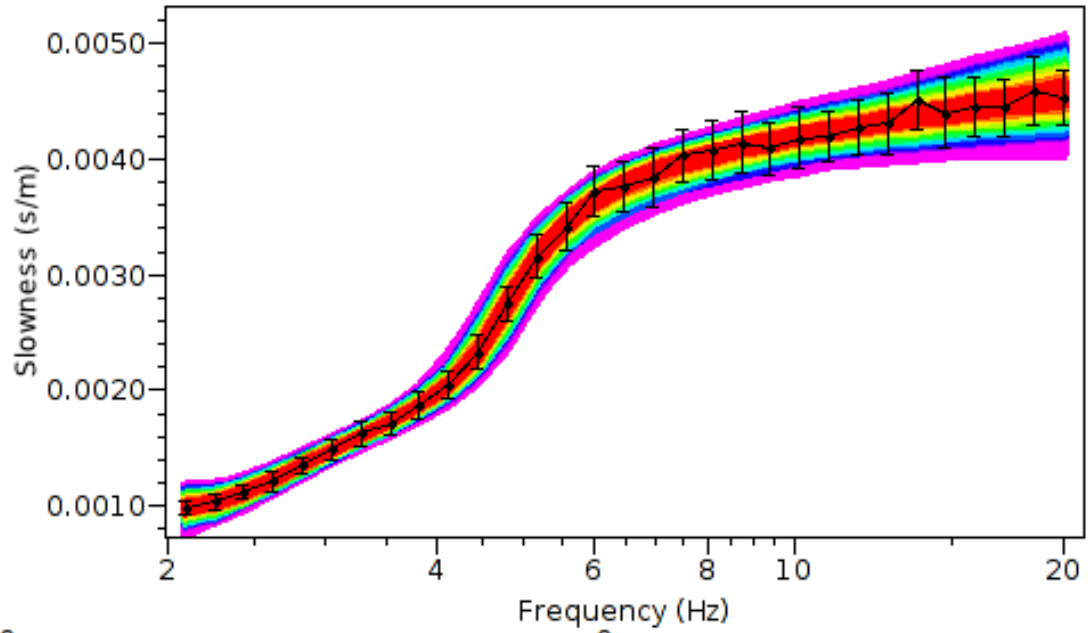
Effects of frequency range

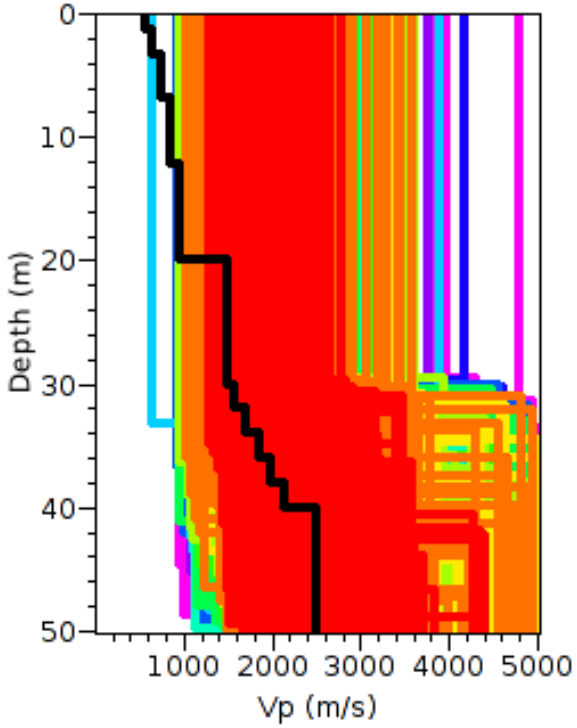
Effects of frequency range

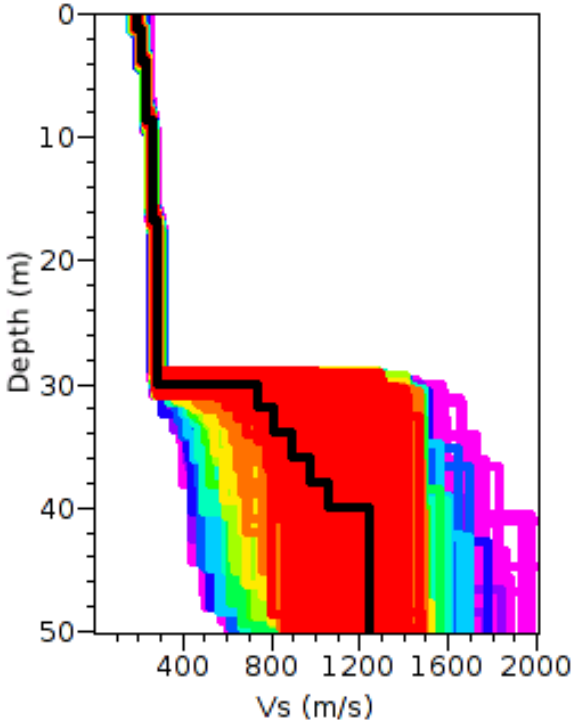


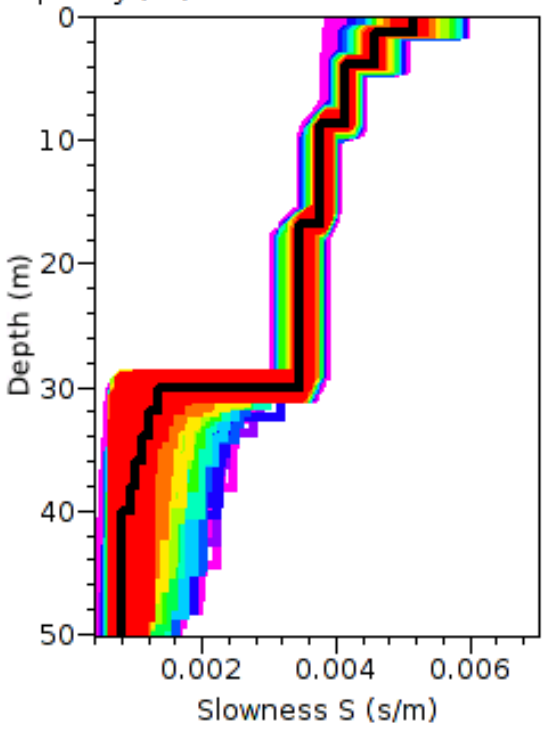
Sharp depth constraint



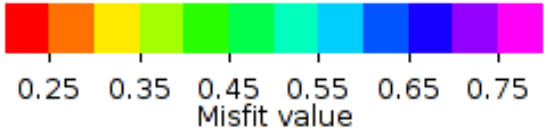


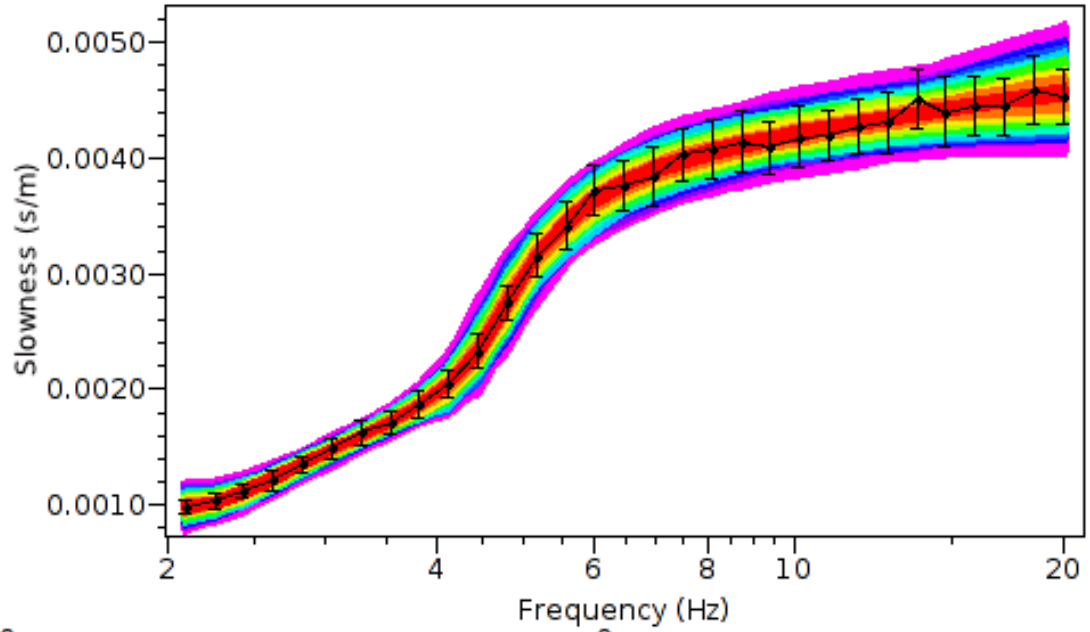


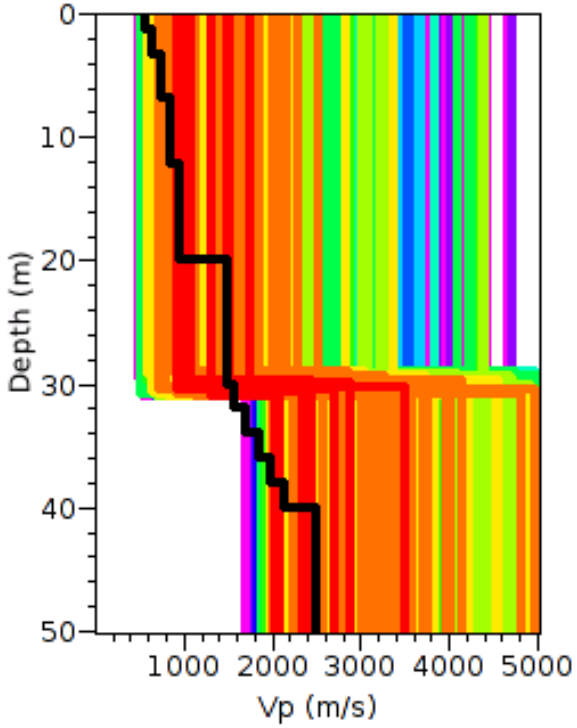


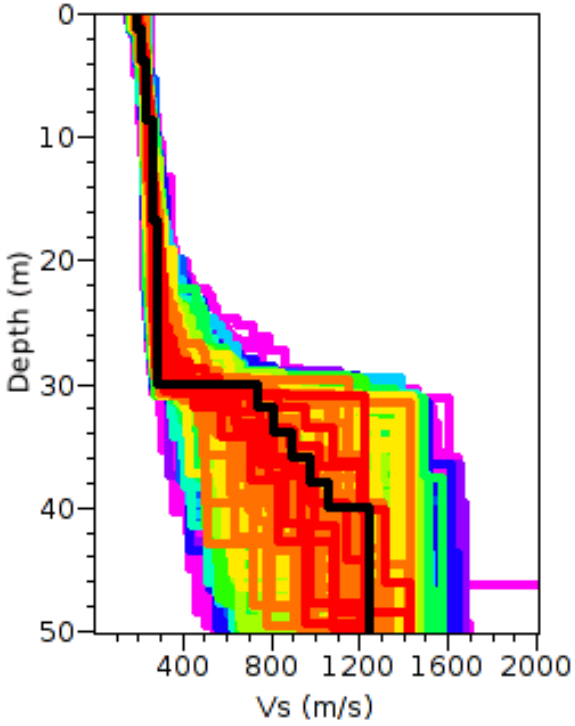


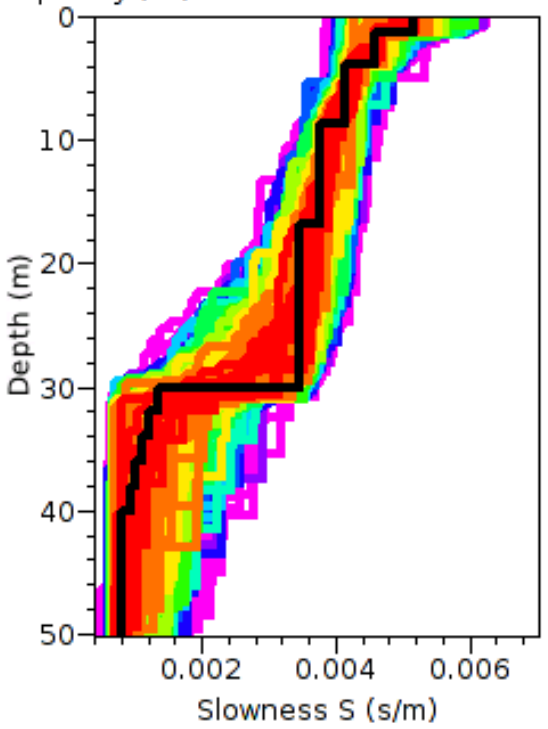
Smooth depth constraint



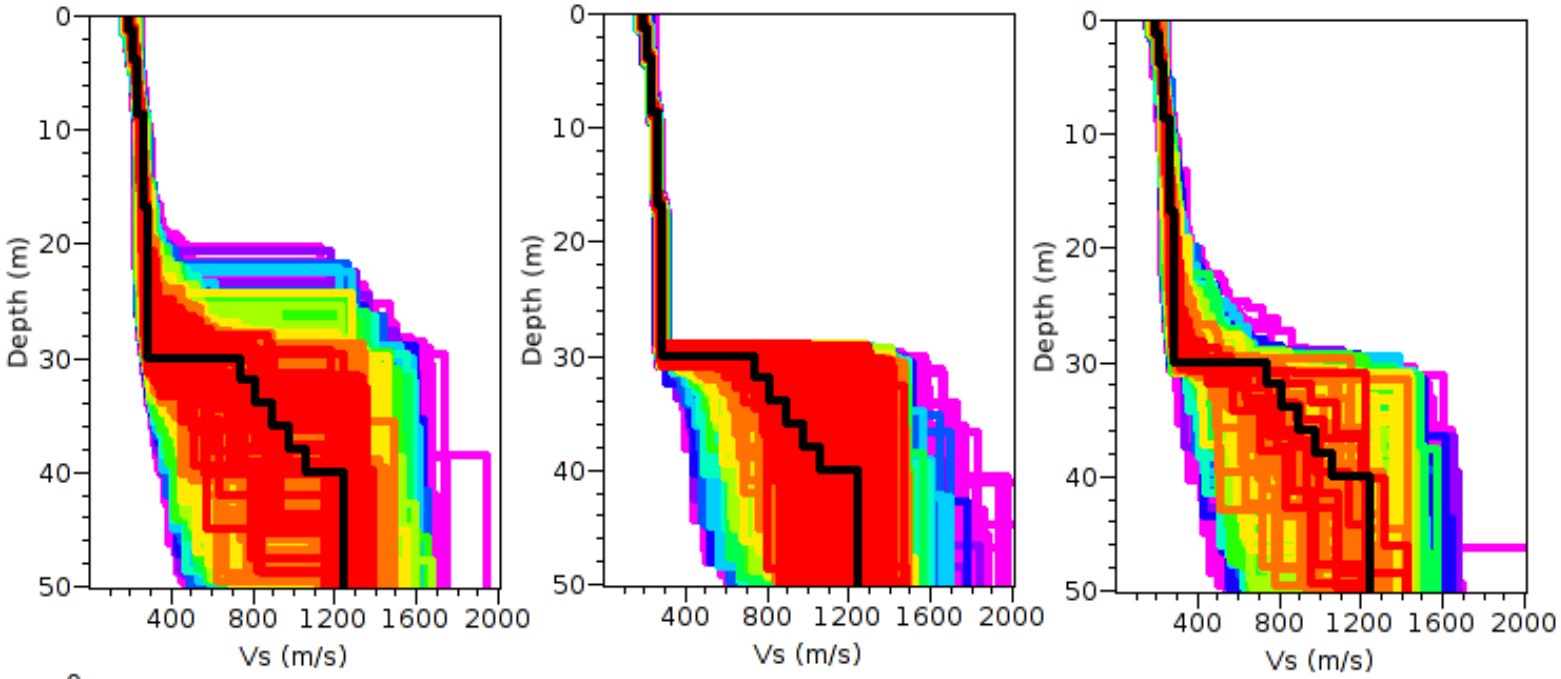


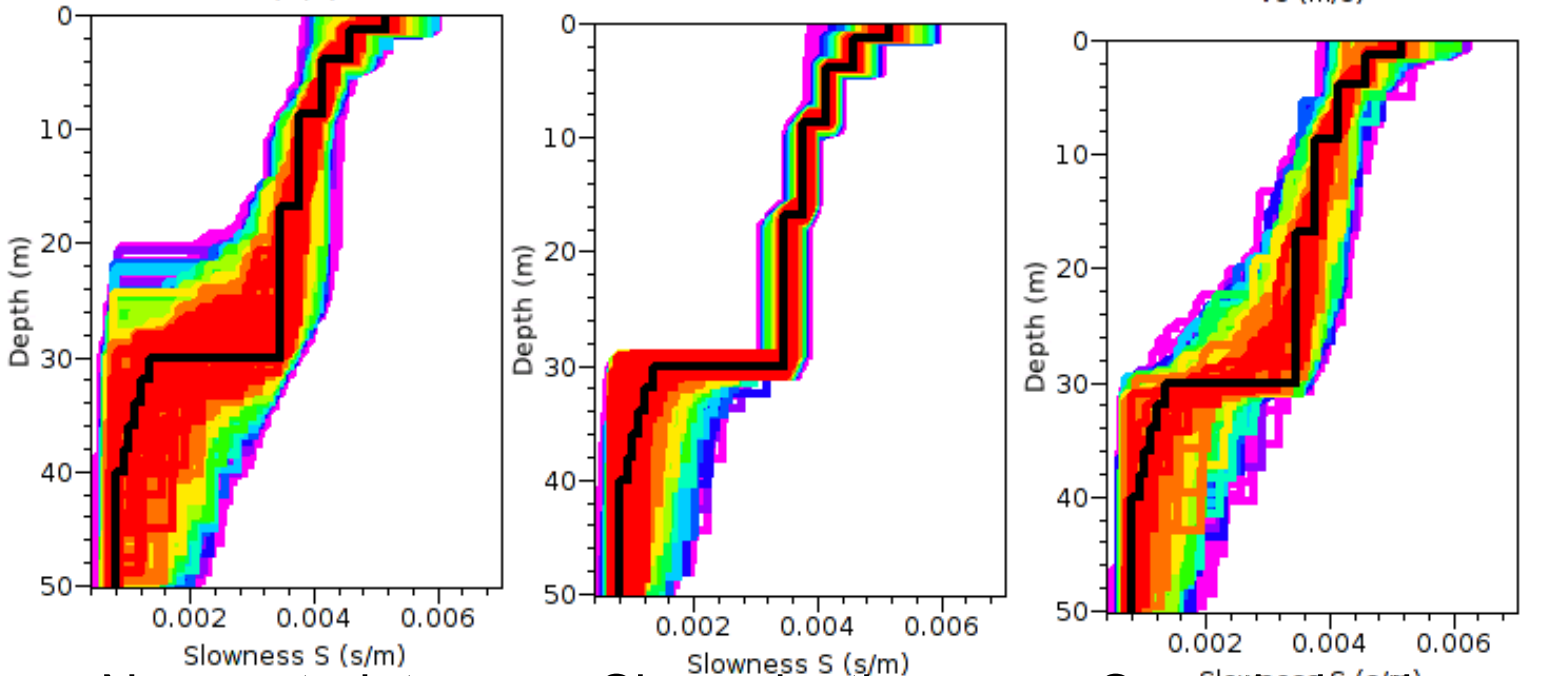






Effects of depth contraint

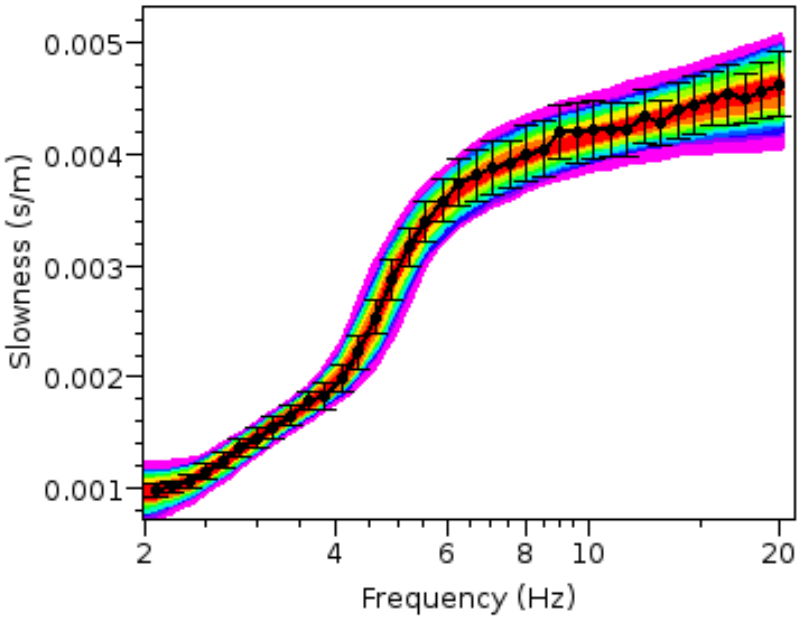


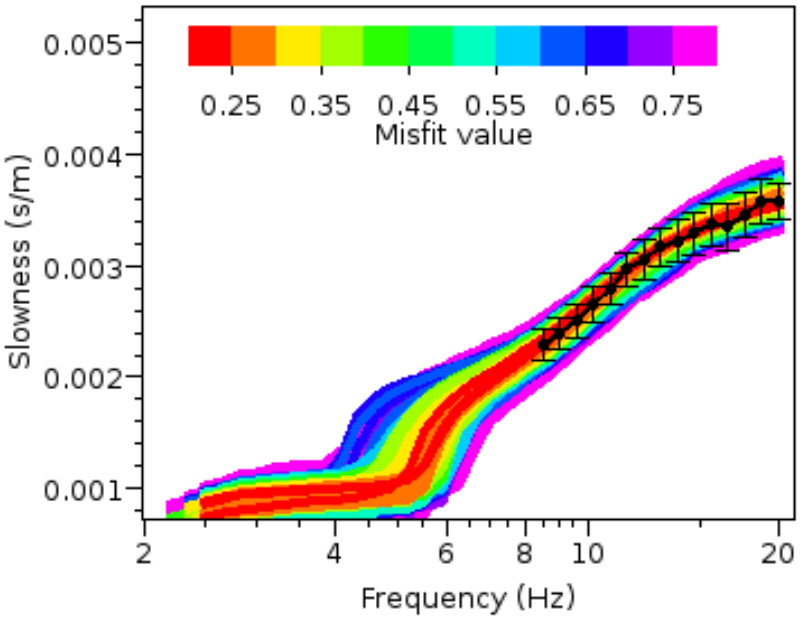


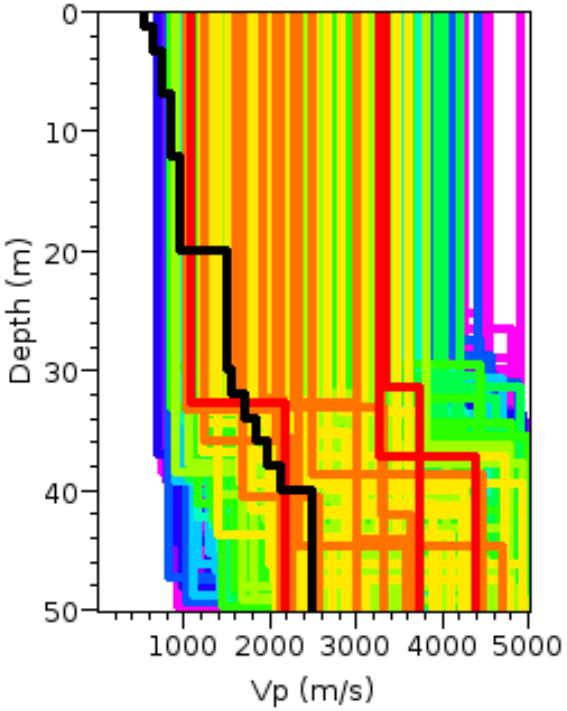
No constraint

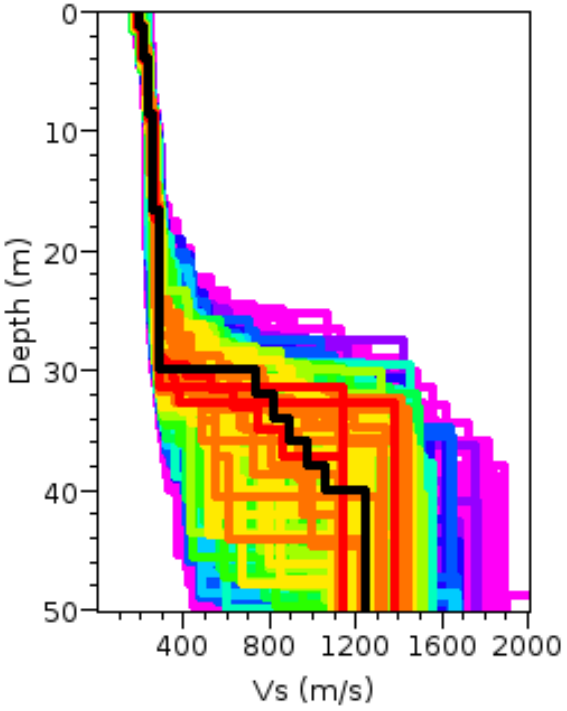
Sharp depth

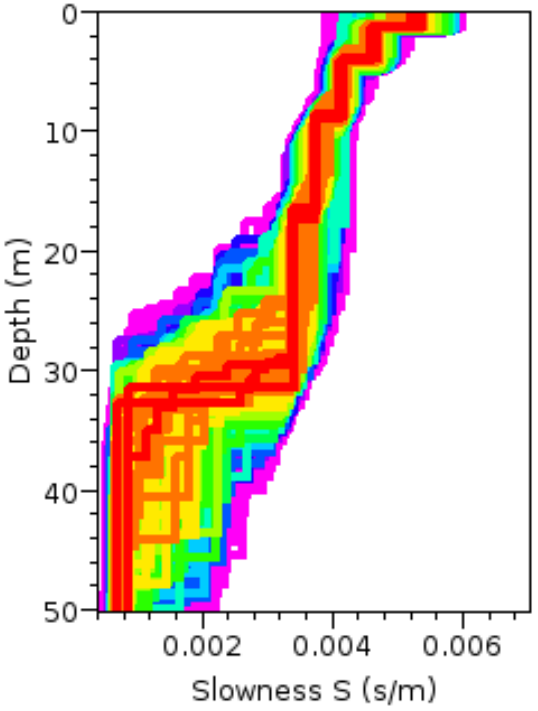
Smooth depth


Slowness (s/m)
0.005
0.004
0.003
0.002
0.001
2
4
6
8
10
20
Frequency (Hz)


0.25
0.35
0.45
0.55
0.65
0.75
Misfit value
Slowness (s/m)
0.005
0.004
0.003
0.002
0.001
2
4
6
8
10
20
Frequency (Hz)

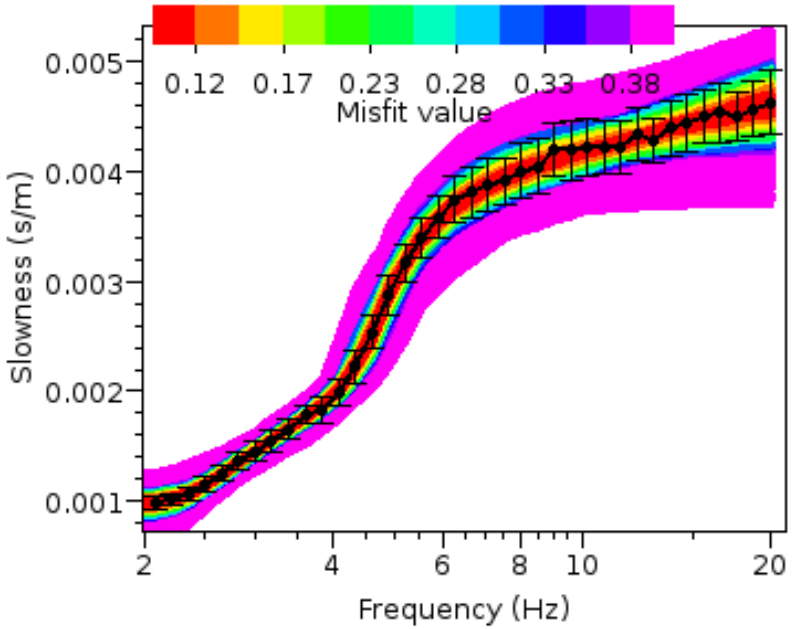

Depth (m)
0
10
20
30
40
50
1000
2000
3000
4000
5000
Vp (m/s)

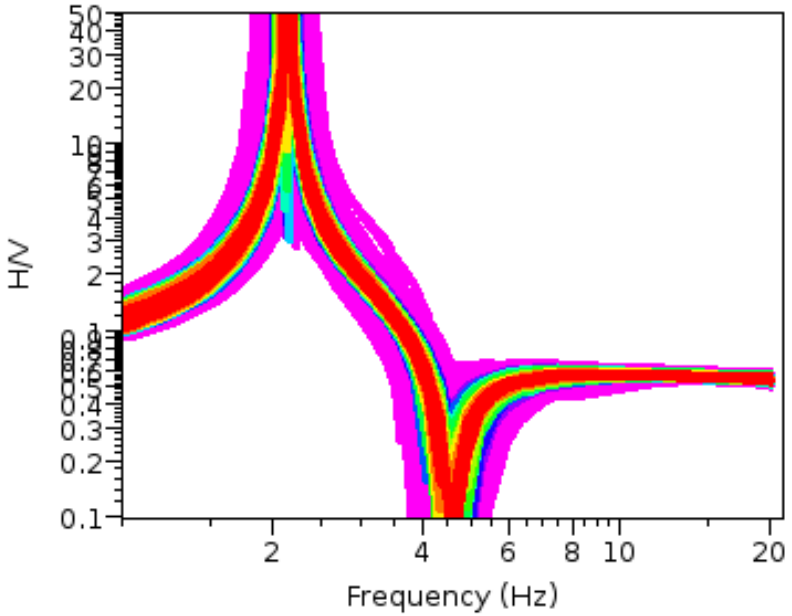

Depth (m)
0
10
20
30
40
50
400
800
1200
1600
2000
Vs (m/s)

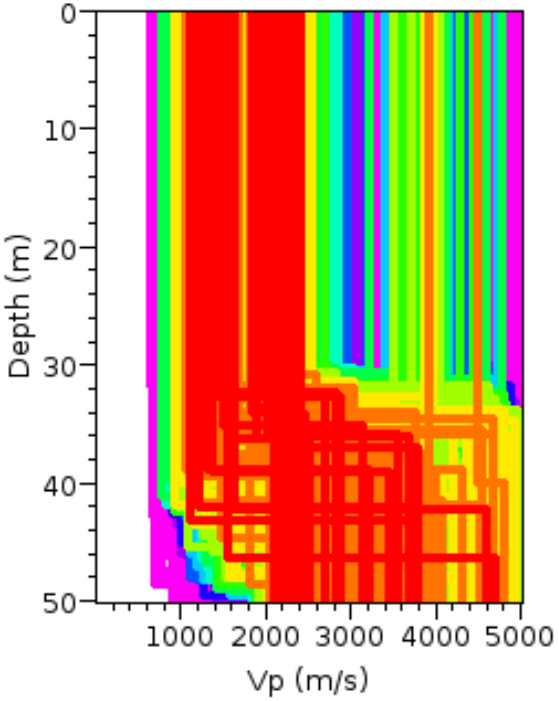

Depth (m)
0
10
20
30
40
50
0.002
0.004
0.006
Slowness S (s/m)

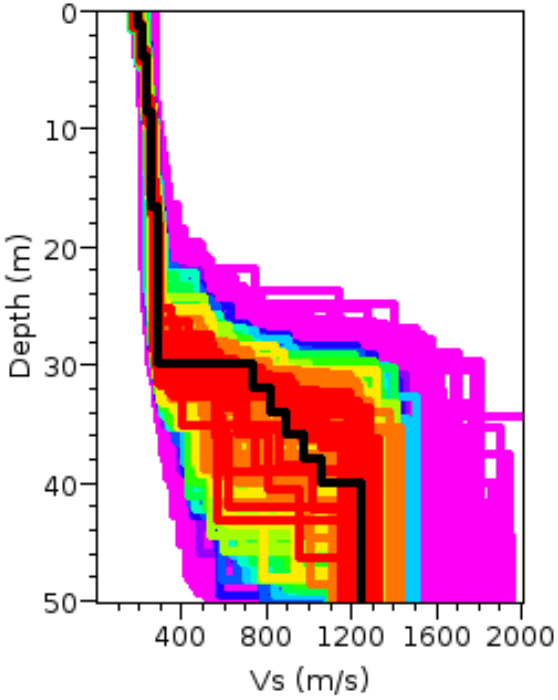
Higher mode

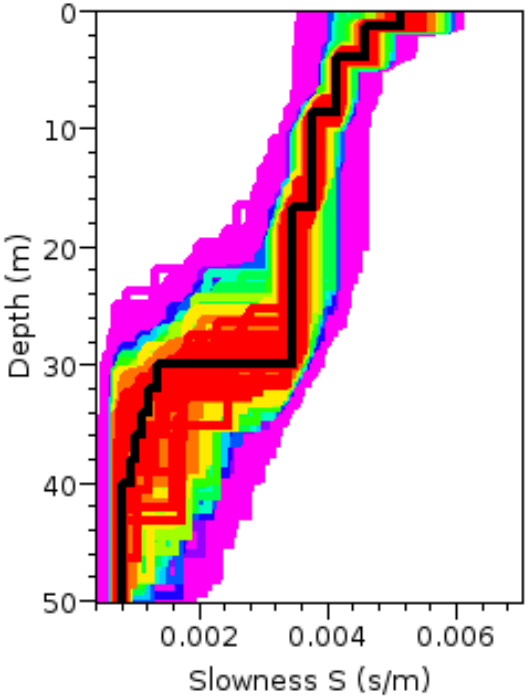
Joint inversion of H/V peak



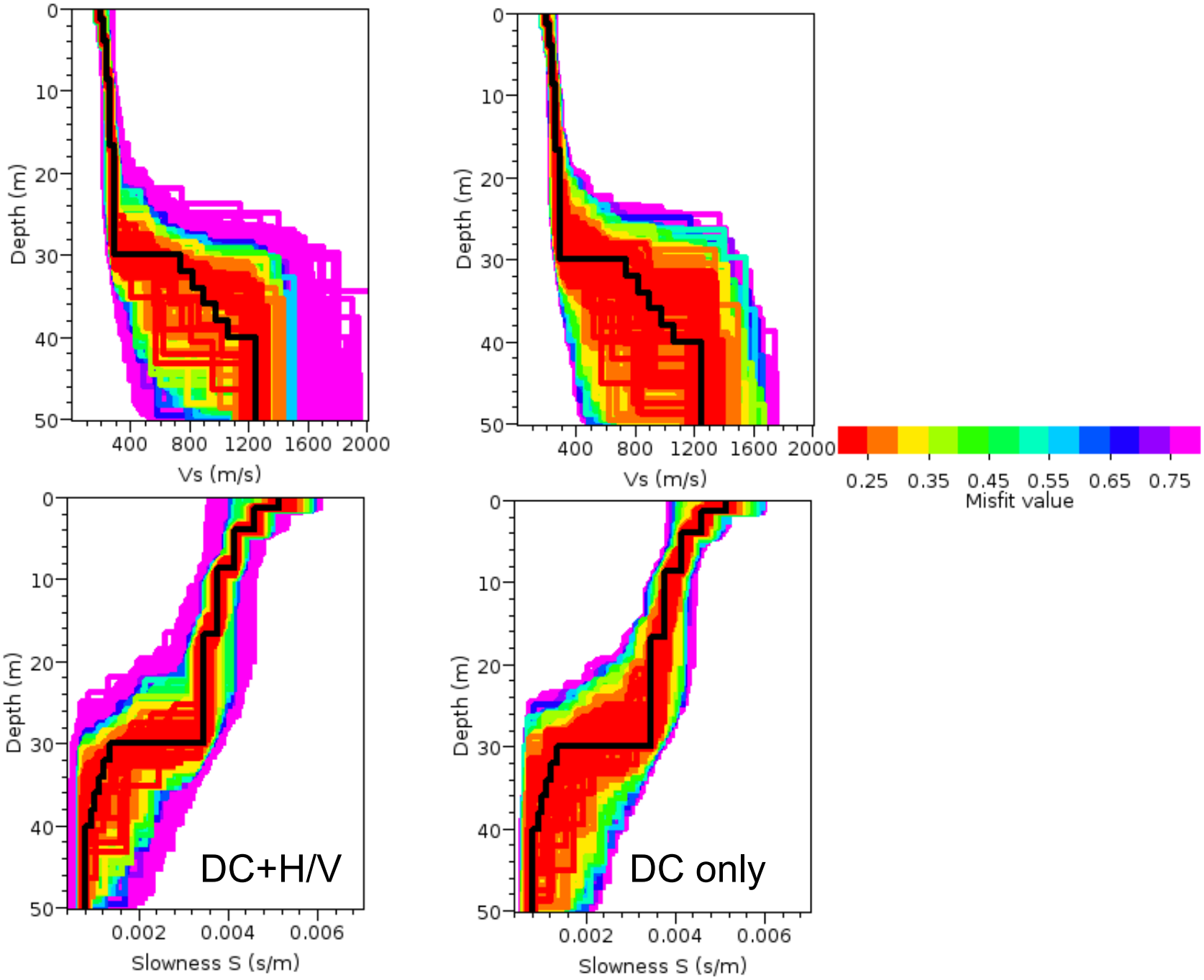




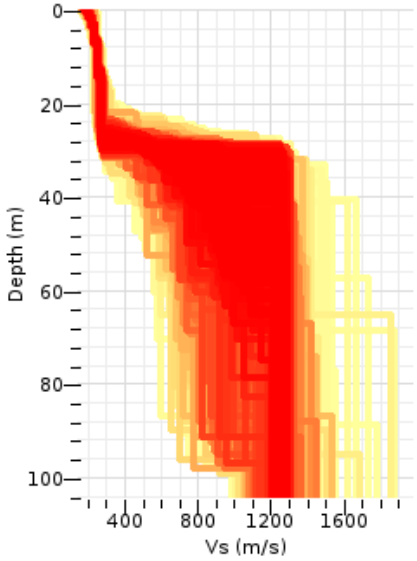




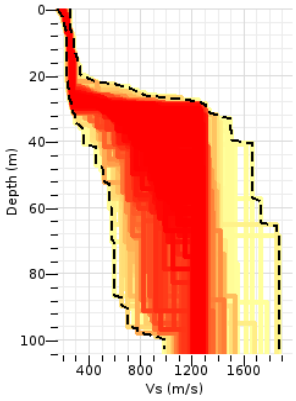
Joint inversion of H/V peak



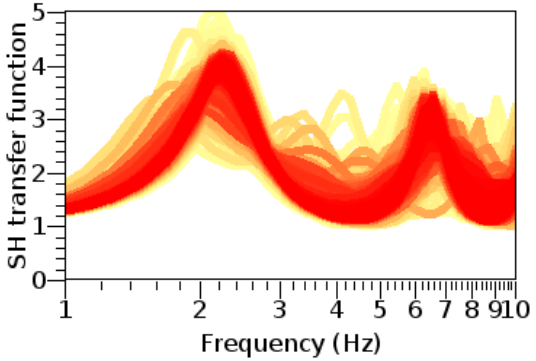
7. What can we do with all inversion solutions?



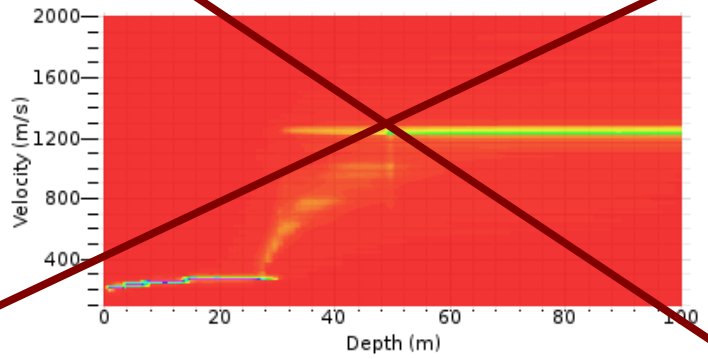
Enveloppe
(min misfit)

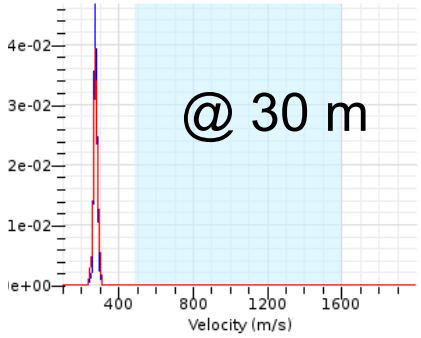


Other computations
with all models



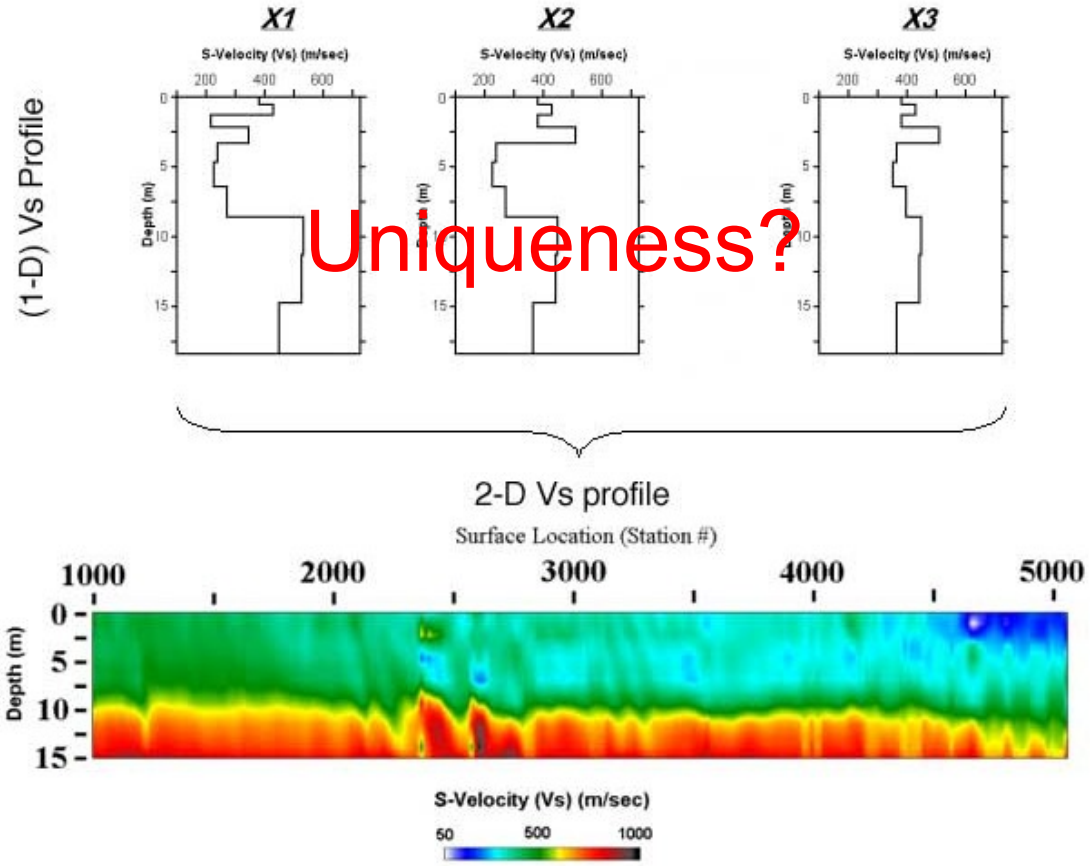
Statistics?





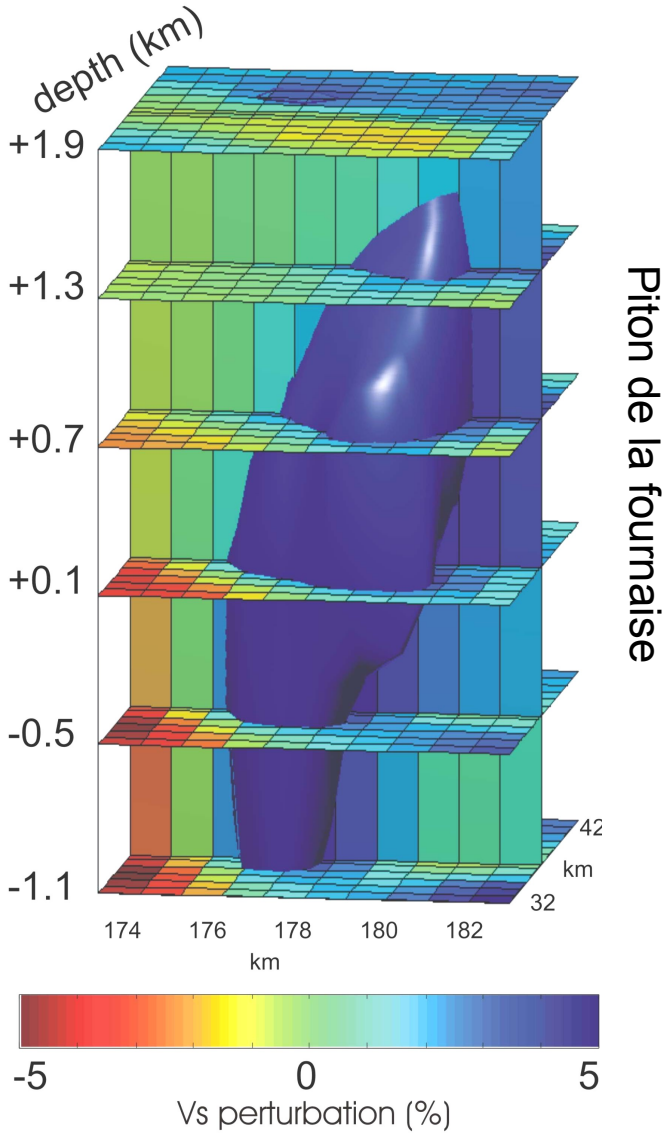
Importance
REQUIRED

2D imaging?



Source: http://www.kgs.ku.edu/software/surfseis

3D Imaging?



D'après Brenguier et al, GRL, 2007

Conclusions

- **Neighborhood Algorithm for parameter spaces with irregular boundaries**
- **Exploration capabilities improved**
- Better exploration means also better data fit
- Less forward computations needed to achieve the same data fit
- Speed improvements (active models)
- Parameter space discretization