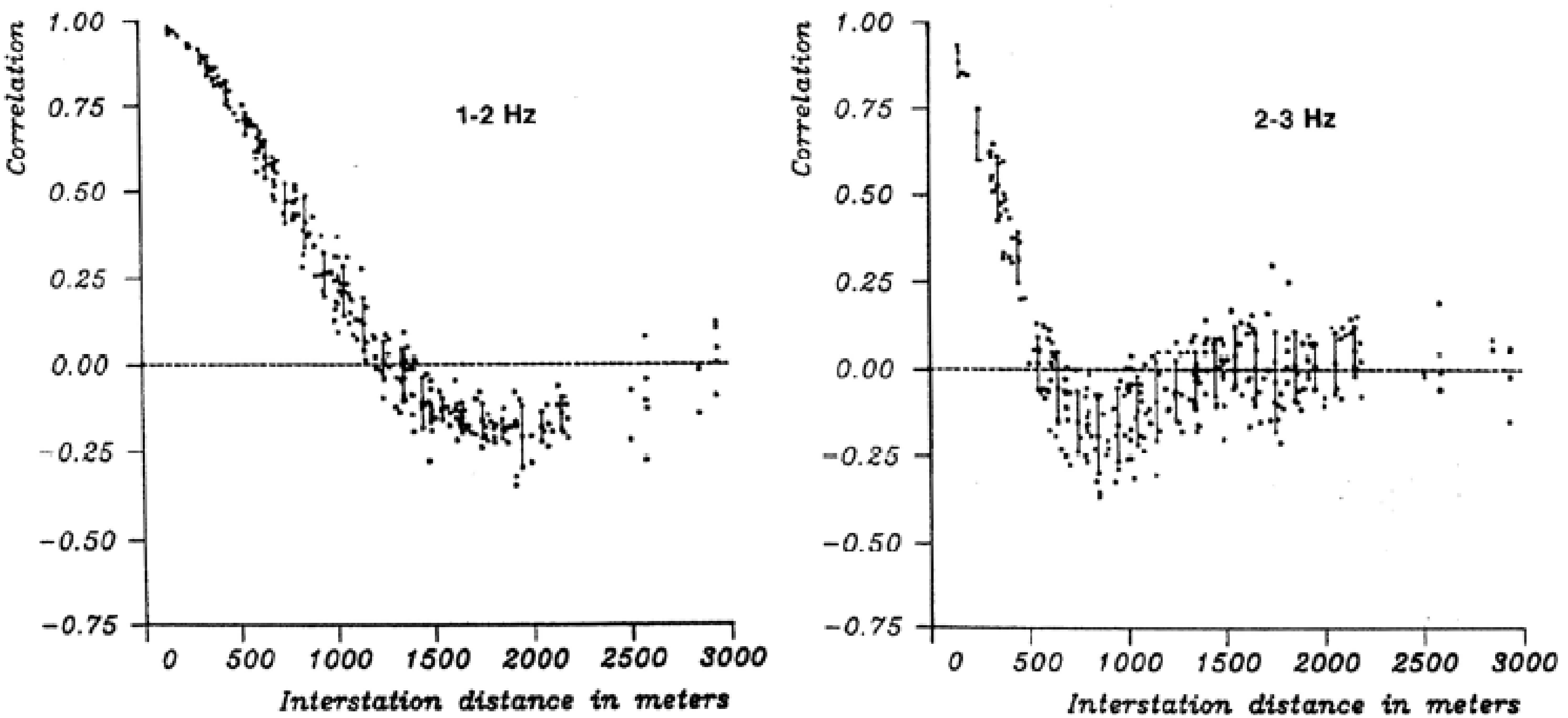
Using Ambient Vibration Array Techniques for Site Characterization

Modified Spatial AutoCorrelation method

(MSPAC)

Februrary 21st-28th 2010, Thessaloniki, Greece

NORES noise correlation analysis ⇨ coherence lengths



30 s long and taken at 05.15 h GMT on day 323 of 1985. Mean values and standard deviations within 100 m distance intervals are plotted on top of the population, except for short and long distances, where the number of correlation values is low.

Mykkeltveit, S., K. Åstebøl, D.J. Dornboos & E.S. Husebye (1983): Seismic array configuration optimization. Bull. Seism. Soc. Am., 73: 173-186.

SPAC (Aki, 1957)

Hypothesis: Stochastic noise wavefield stationary in both time and space

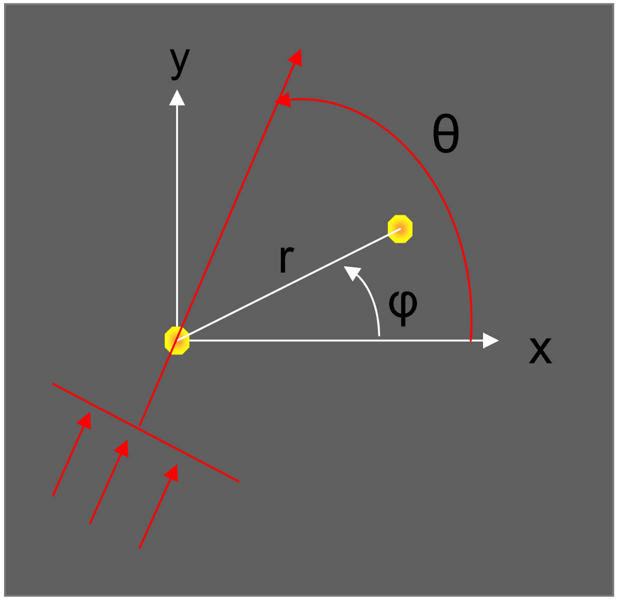
Spatial correlation function

$$\phi(\mathbf{r},\varphi)=\frac{1}{\mathbf{T}}\int_0^{\mathbf{T}}\mathbf{u}(\mathbf{x},\mathbf{y},\mathbf{t})\mathbf{u}^*(\mathbf{x}+\mathbf{r}\cos\varphi,\mathbf{y}+\mathbf{r}\sin\varphi,\mathbf{t})\mathbf{dt}$$

Using relation between spectrum in time and spectrum in frequency, for one source with propagation azimuth θ

$$\phi(r,\varphi)=\frac{1}{\pi}\int_0^{\infty}\Phi(\omega)\cos\left[\frac{\omega r}{c(\omega)}\cos(\theta-\varphi)\right]d\omega$$

cross-spectrum



Narrow band filtering around w_o

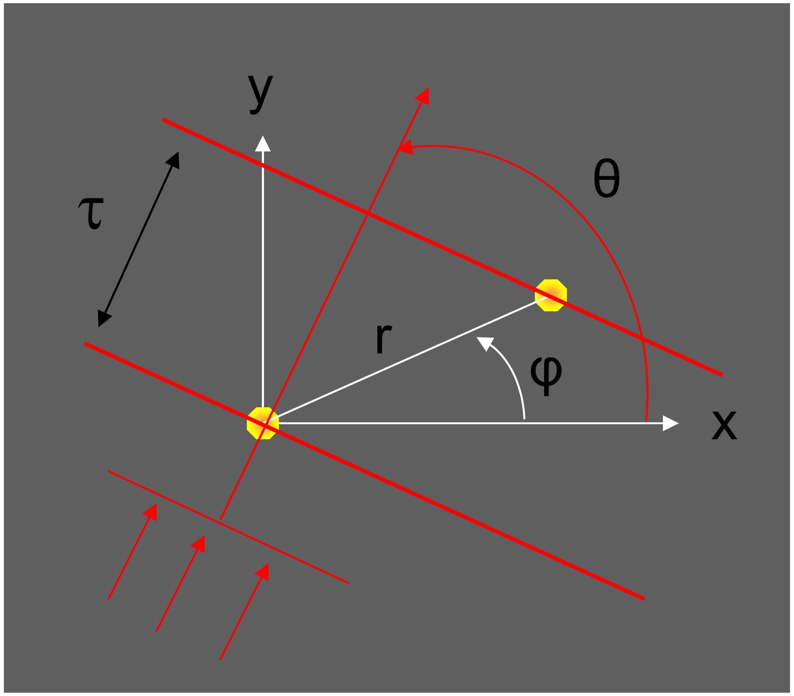
$$\Phi(\omega)=\Phi(\omega_0)\delta(\omega-\omega_0)$$

$$\phi(r,\varphi,\omega_0)=\frac{1}{\pi}\Phi(\omega_0)\cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

Correlation coefficient

$$\rho(r,\varphi,\omega_0)=\frac{\phi(r,\varphi,\omega_0)}{\phi(0,\varphi,\omega_0)}=\cos\left[\frac{\omega_0\mathbf{r}}{\mathbf{c}(\omega_0)}\cos(\theta-\varphi)\right]$$

Correlation coefficient



$$\rho(\mathbf{r},\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

$$\tau = \frac{\mathbf{r}}{\mathbf{c}(\omega_0)}\cos(\theta-\varphi)$$

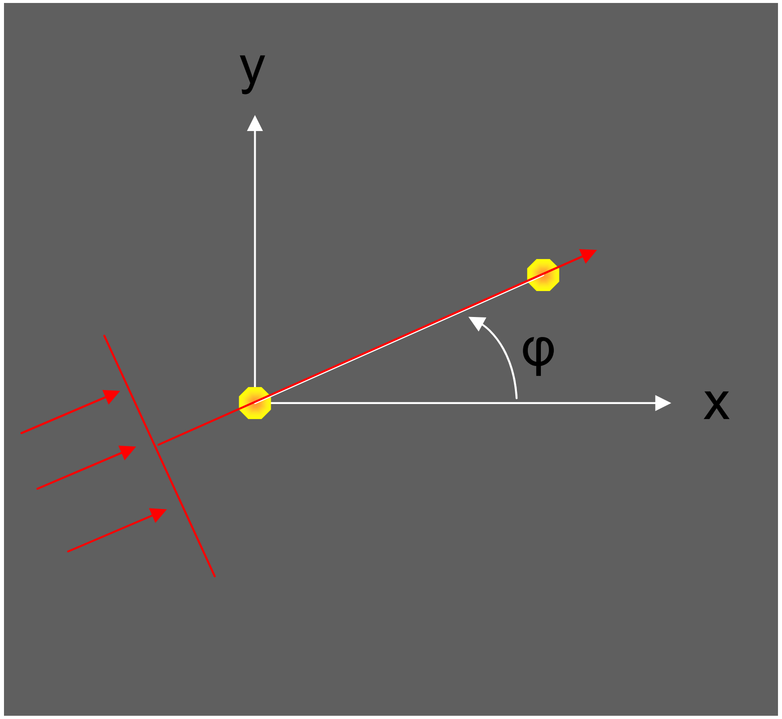
Time delay

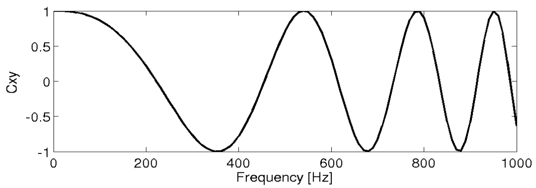
$$\rho(\mathbf{r},\varphi,\omega_0) = \cos[\omega_0 \tau]$$

Correlation coefficient

$\theta - \varphi = 0$

$c(\omega)$

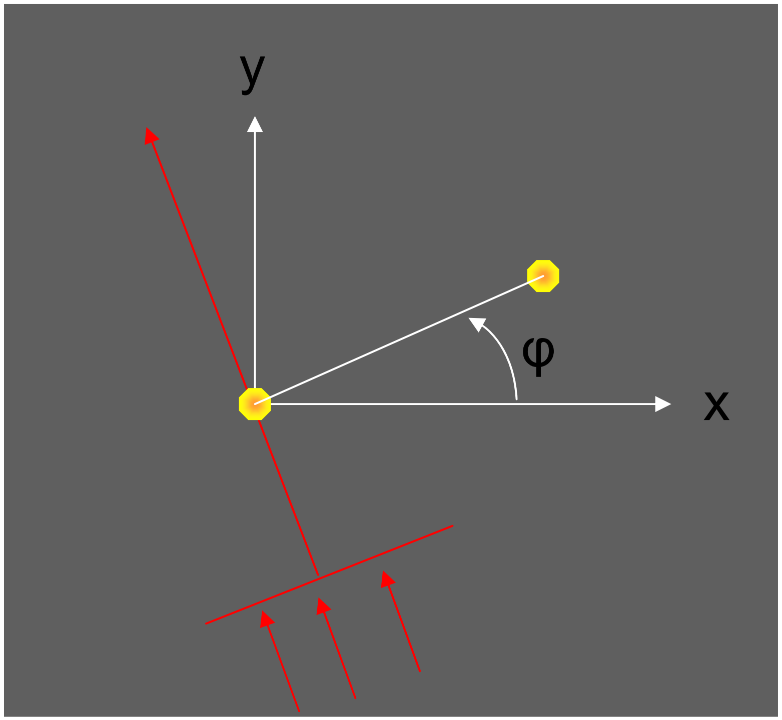




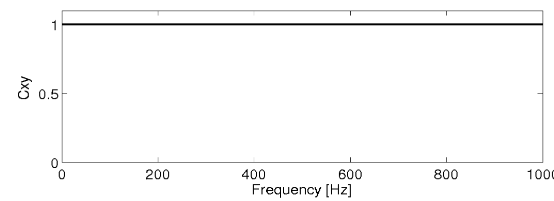
$$\rho(\mathbf{r}, \varphi, \omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta - \varphi)\right]$$

Correlation coefficient

$c(\omega)$

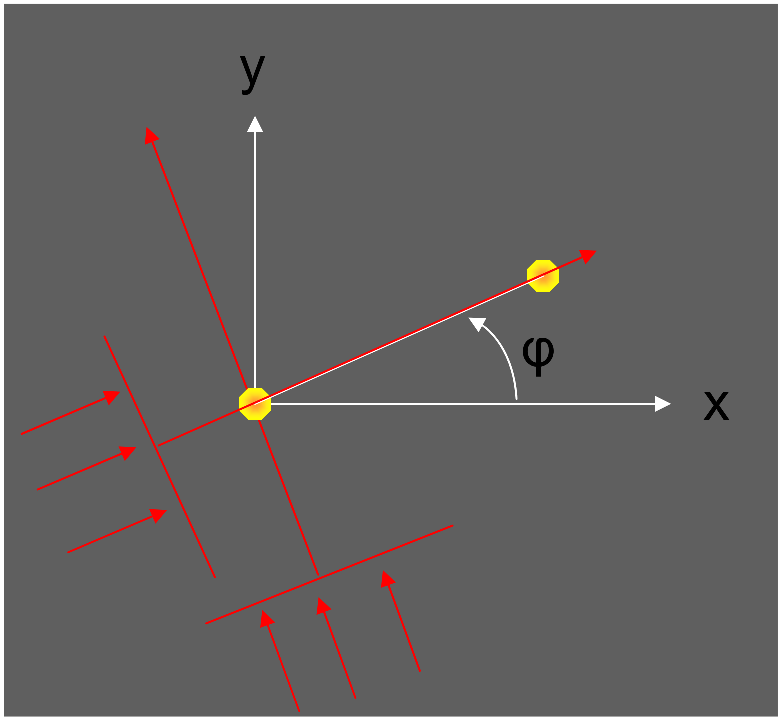


$\theta - \varphi = \pi/2$

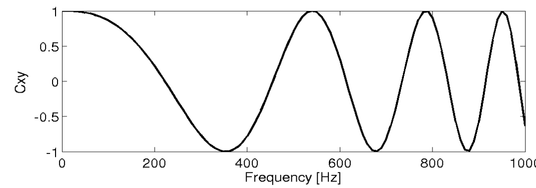


$$\phi(r,\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

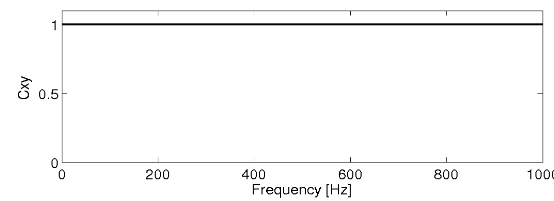
Correlation coefficient

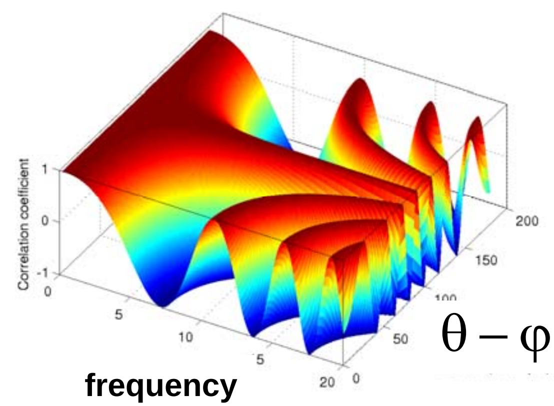


$\theta - \varphi = 0$ c(ω)



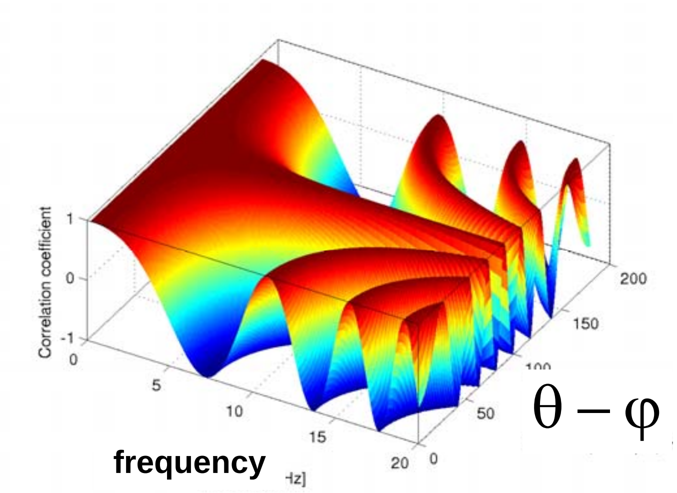
$\theta - \varphi = \pi/2$





$$\phi(r,\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

Correlation coefficient



$$\phi(r,\varphi,\omega_0) = \cos\left[\frac{\omega_0 r}{c(\omega_0)}\cos(\theta-\varphi)\right]$$

For a single plane wave propagating with back-azimuth θ (except $\theta - \varphi = \pi/2$) and two sensors, $c(\omega_0)$ can be estimated

For ambient noise however, multiple sources (several θ) ….

Average spatial aucorrelation coefficient

For a large number of azimuthaly distributed sources

Azimuthal averaging of spatial autocorrelation coefficients

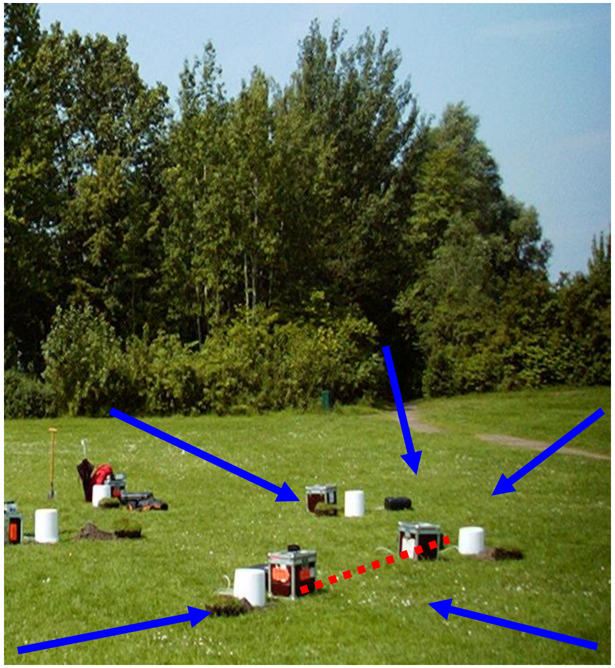
$$\bar{\rho}(\mathbf{r},\omega_0) = \frac{1}{\pi}\int_0^{\pi} \rho(\mathbf{r},\varphi,\omega_0)\mathbf{d}(\theta-\varphi)$$

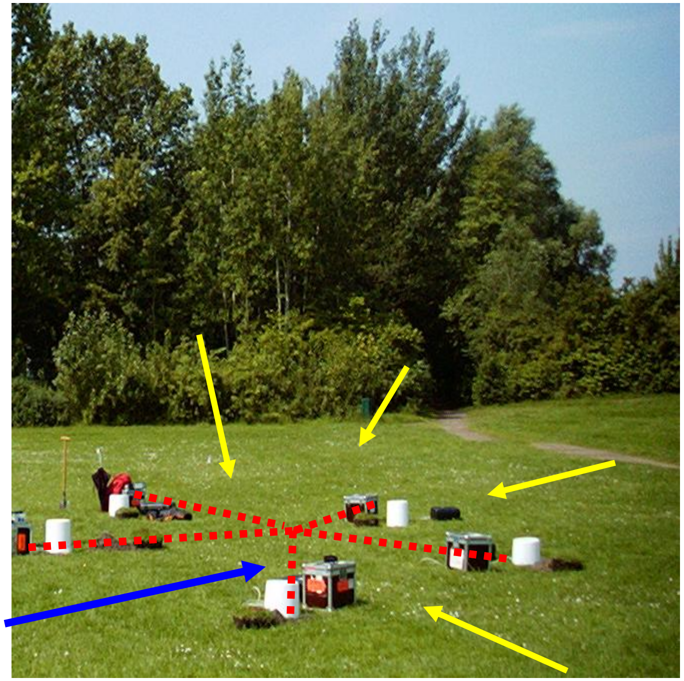
$$\bar{\rho}(r,\omega_0) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$

$$J_0(x) = 1/\pi \int_0^{\pi} \cos(x\cos(\varphi))\, d\varphi$$

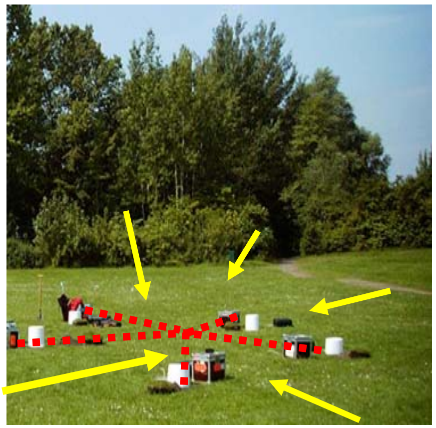
Average spatial aucorrelation coefficient (N-sensors case)

In ambient noise, number and spatial distribution of sources are unknowns …





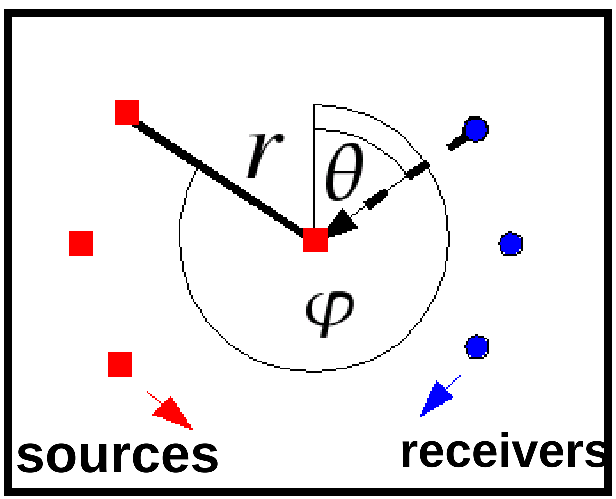
Average spatial aucorrelation coefficient (N-sensors, circular arrays)

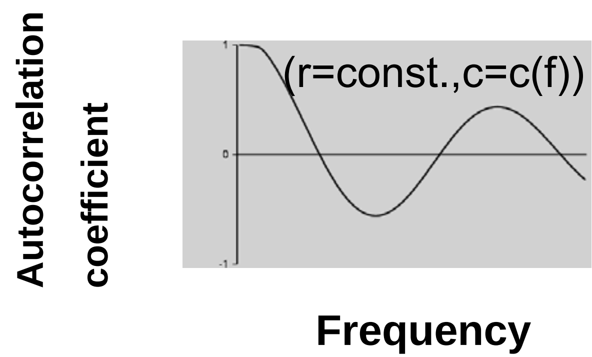


Azimuthal averaging of spatial autocorrelation coefficients

$$\bar{\rho}(\mathbf{r},\omega_0) = \frac{1}{\pi}\int_0^{\pi} \rho(\mathbf{r},\varphi,\omega_0)\mathbf{d}(\theta-\varphi)$$

$$\bar{\rho}(r,\omega_0) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$

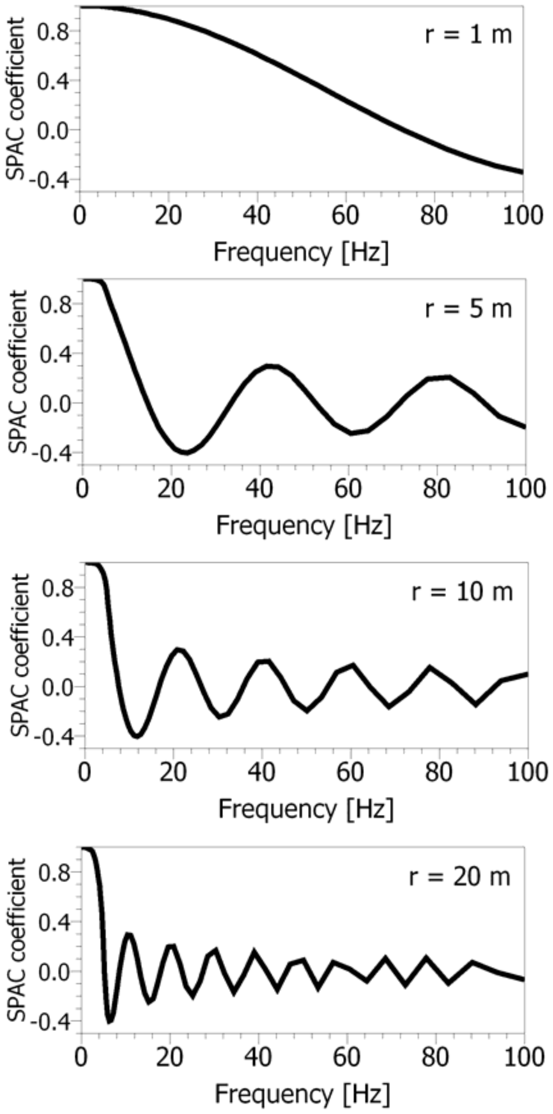




SPAC coefficient

$$\bar{\rho}(r,\omega_0) = J_0\left(2\pi \frac{r}{\lambda(\omega_0)}\right) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$

Thickness (m)	Vp (m/s)	Vs (m/s)
20	1000	200
-	2000	1000

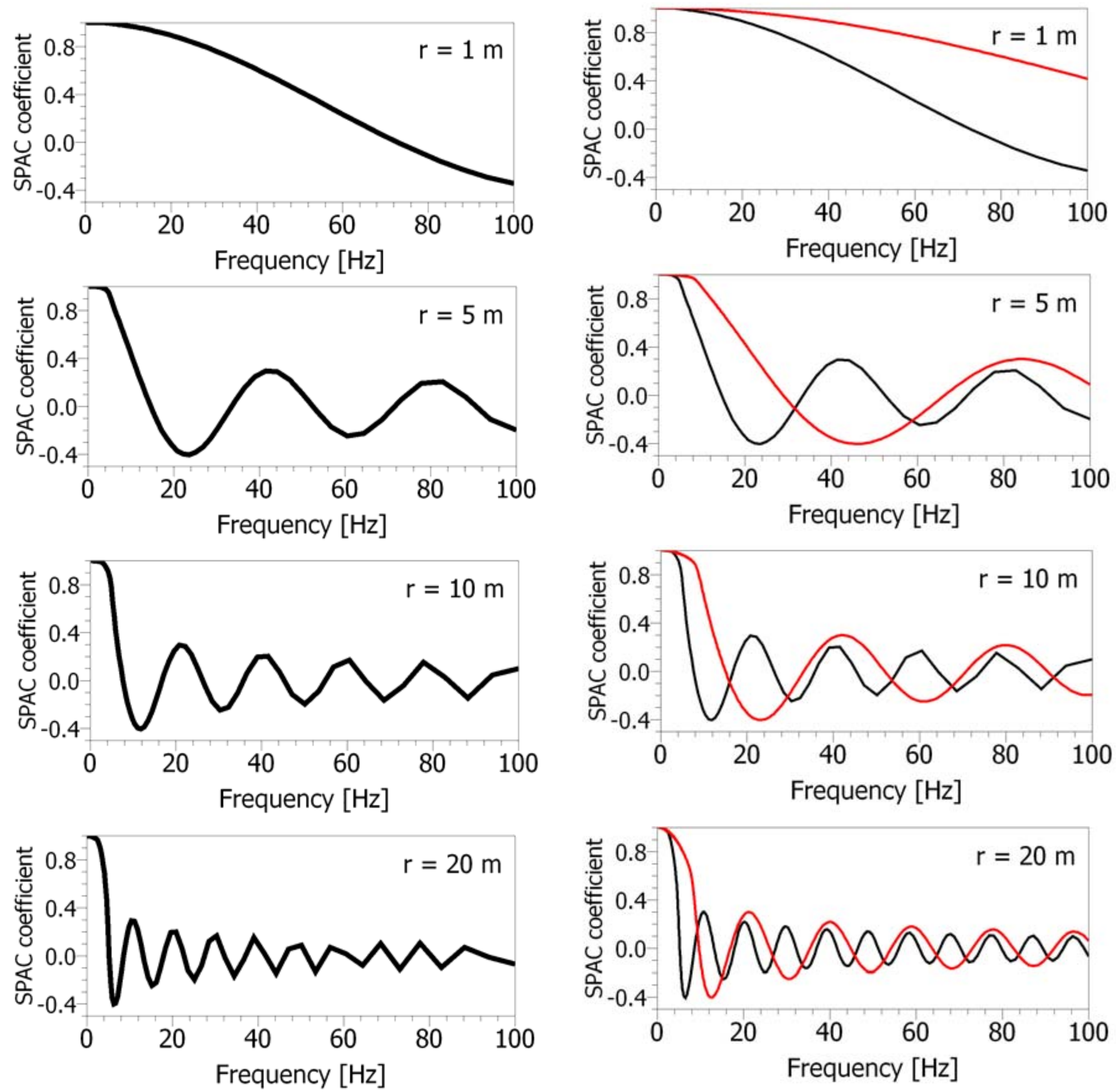


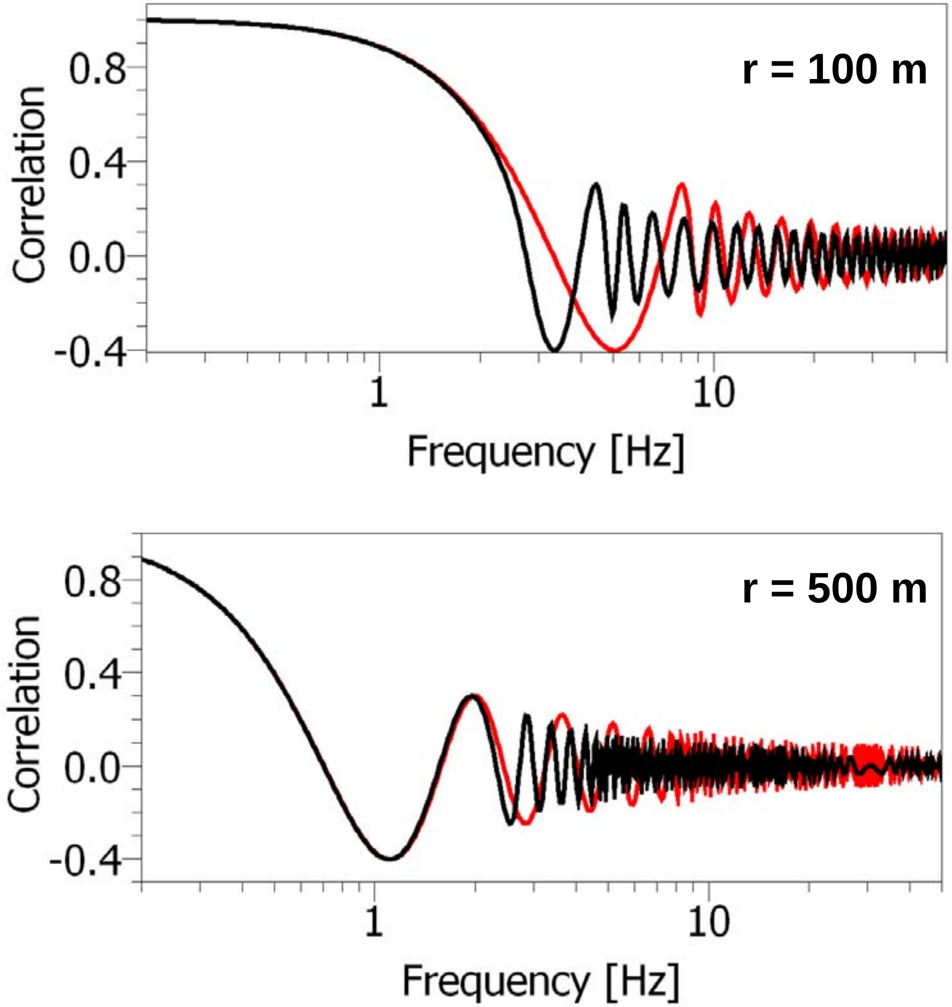
SPAC coefficient

$$\bar{\rho}(r,\omega_0)=J_0\left(2\pi\frac{r}{\lambda(\omega_0)}\right)=J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$

Thickness (m)	Vp (m/s)	Vs (m/s)
20	1000	200
-	2000	1000

Thickness (m)	Vp (m/s)	Vs (m/s)
20	1000	400
-	2000	1000

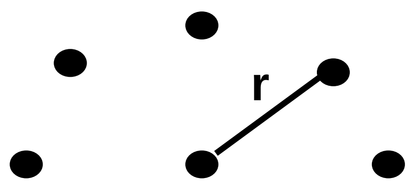




r = 100 m
0.8
0.4
0.0
-0.4
Correlation
1
10
Frequency [Hz]
r = 500 m
0.8
0.4
0.0
-0.4
Correlation
1
10
Frequency [Hz]

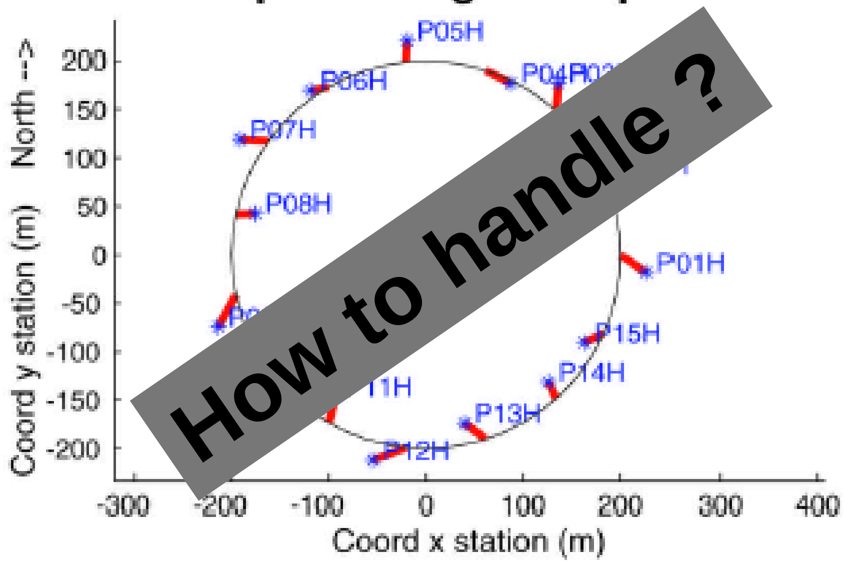
MSPAC (Bettig et al., 2001): extension to arbitary array configurations

SPAC (Aki, 1957)

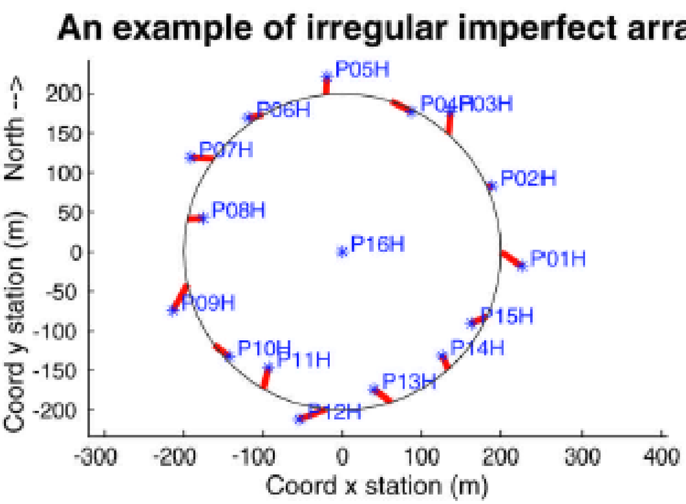


$$\overline{\rho}(r,\omega_0) = J_0\left(\frac{\omega_0 r}{c(\omega_0)}\right)$$

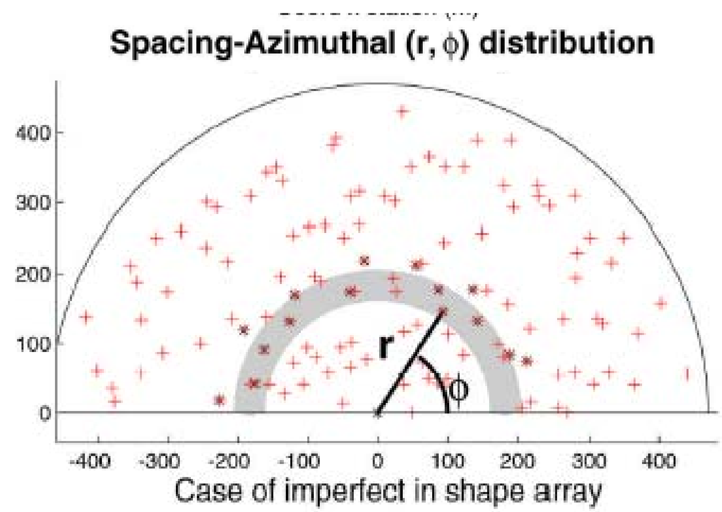
An example of irregular imperfect array



MSPAC (Bettig et al., 2001): extension to arbitary array configurations



co-arrays

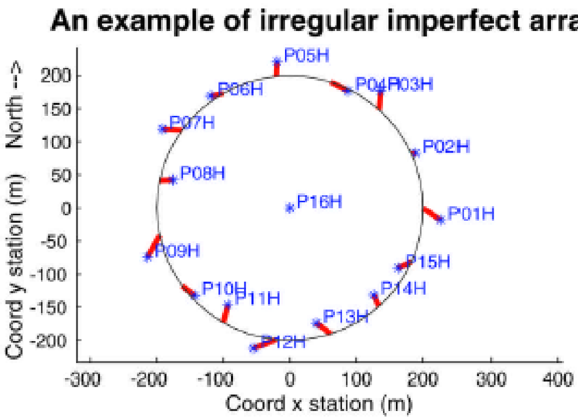


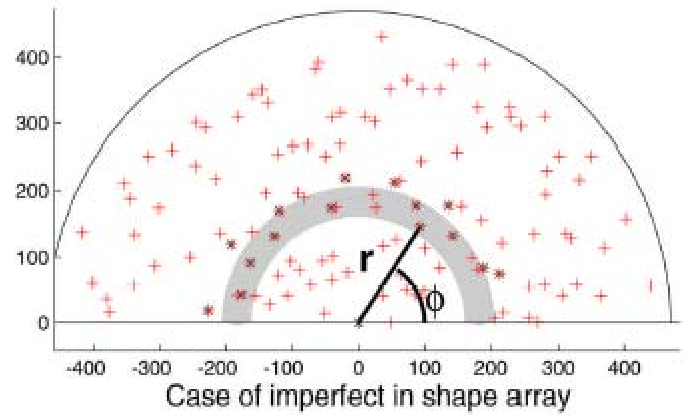
Computation of azimuthal and **radial** averaged spatial autocorrelation coefficients

$$\overline{\rho_{z,r1,r2}}(\omega_0)=\frac{2}{r_2^2-r_1^2}\cdot\frac{c_R(\omega_0)}{\omega_0}\cdot\left[r\cdot J_1\left(\frac{\omega_0\cdot r}{c_R(\omega_0)}\right)\right]_{r_1}^{r_2}$$

With r_1 and r_2 being the inner and outer radius of the ring, respectively

MSPAC vs. SPAC: advantages and drawbacks





Advantages

- in urban areas: easier arrays realisation
- analysis of data sets suitable/optimized for FK
- broader frequency/wavelength range than SPAC (only 1 ring)

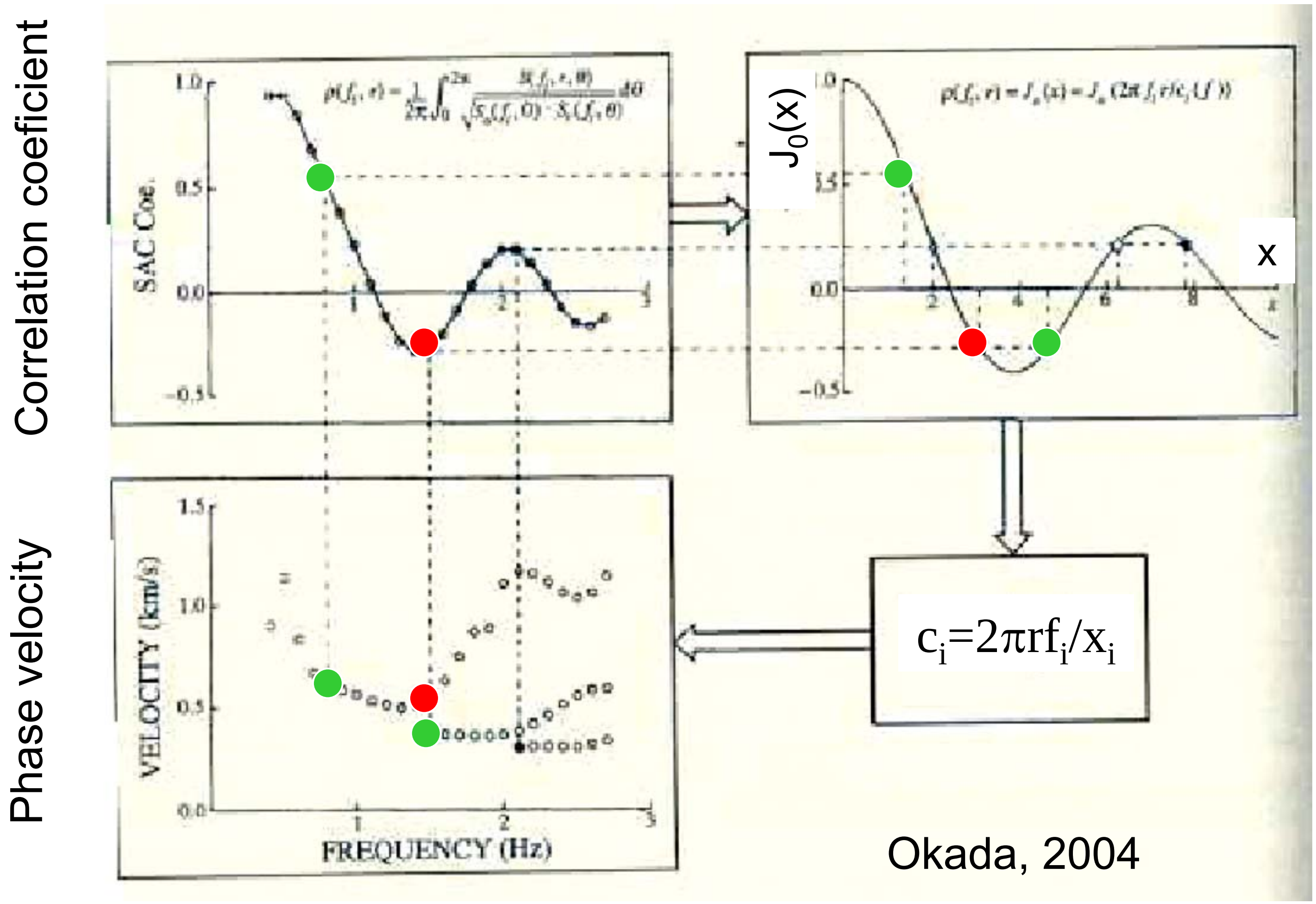
Disavantages

- less « precise » estimation than SPAC

Estimation of phase velocity

$$\bar{\rho}(r,\omega) = J_0\left(\frac{\omega r}{c(\omega)}\right)$$

measure target

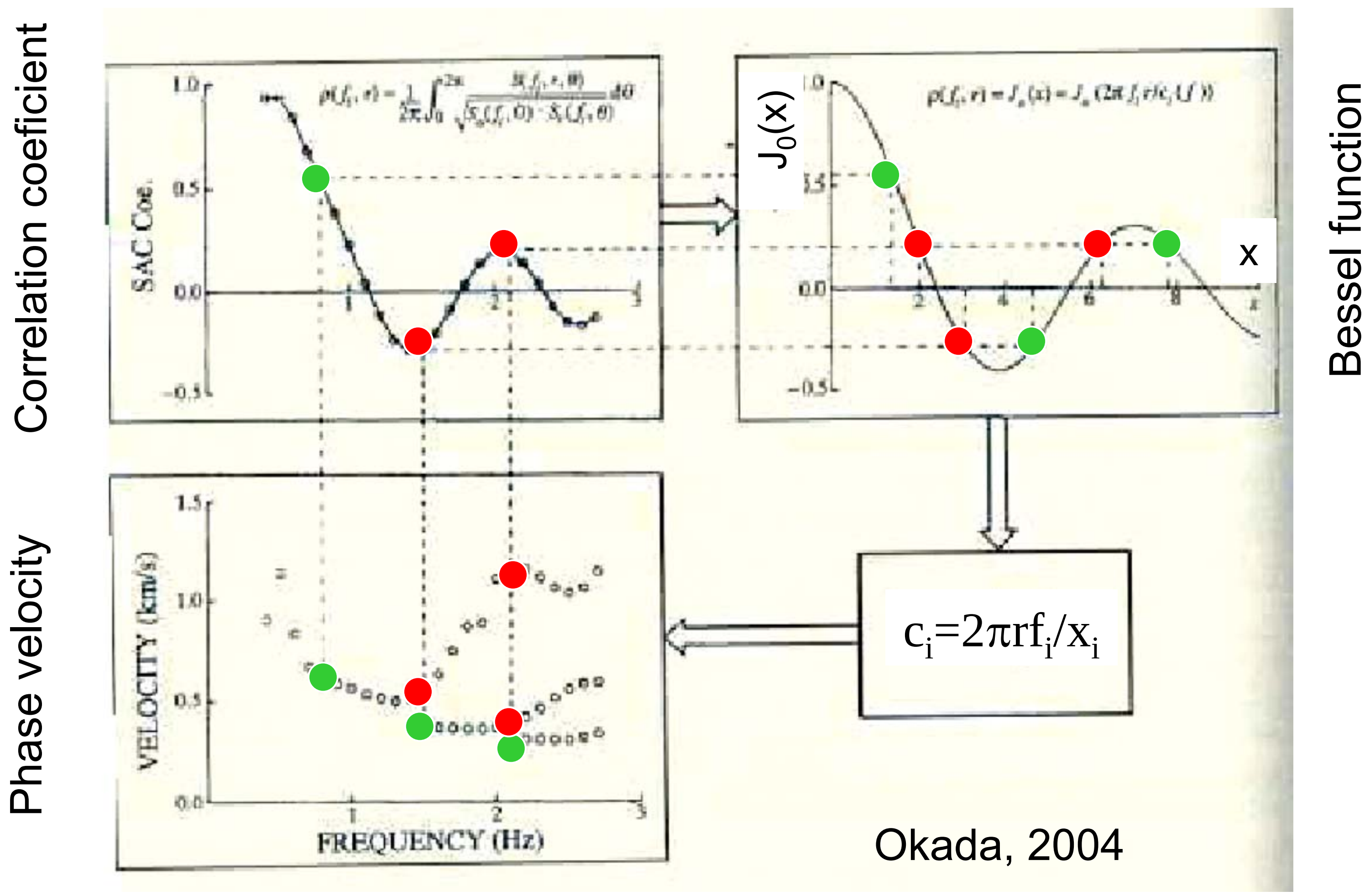


Okada, 2004

Estimation of phase velocity

$$\overline{\rho}(r,\omega) = J_0\left(\frac{\omega r}{c(\omega)}\right)$$

measure   target



Building a broad-band dispersion curve from SPAC:

Resolution and non-uniqueness issues

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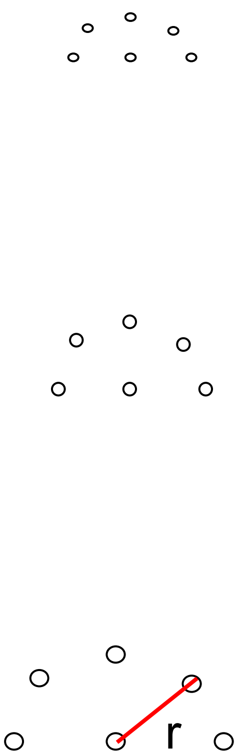
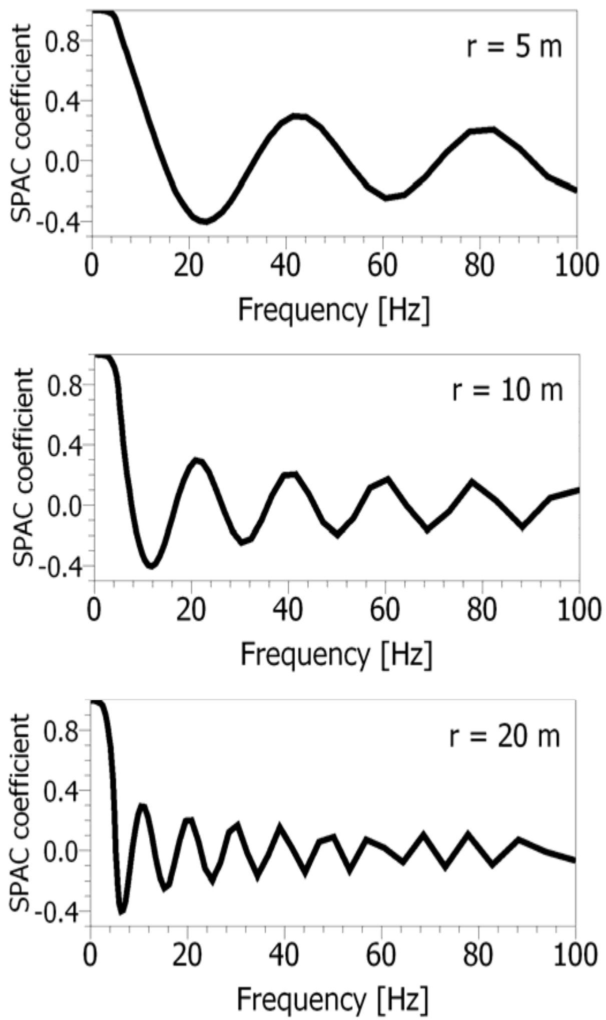
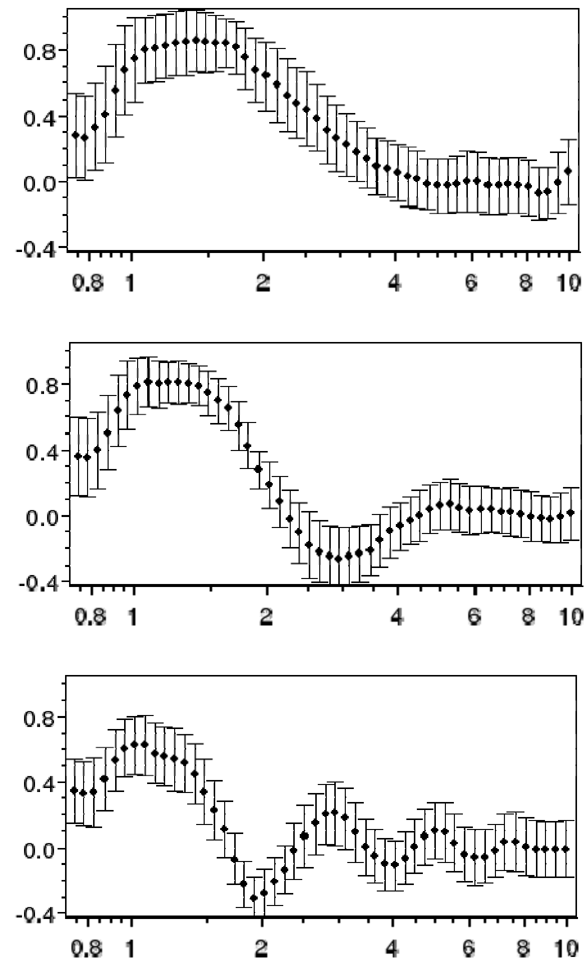
Spatial autocorrelation coefficient for different radius : resolution and non-uniqueness



Theory



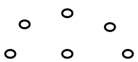
« real life »

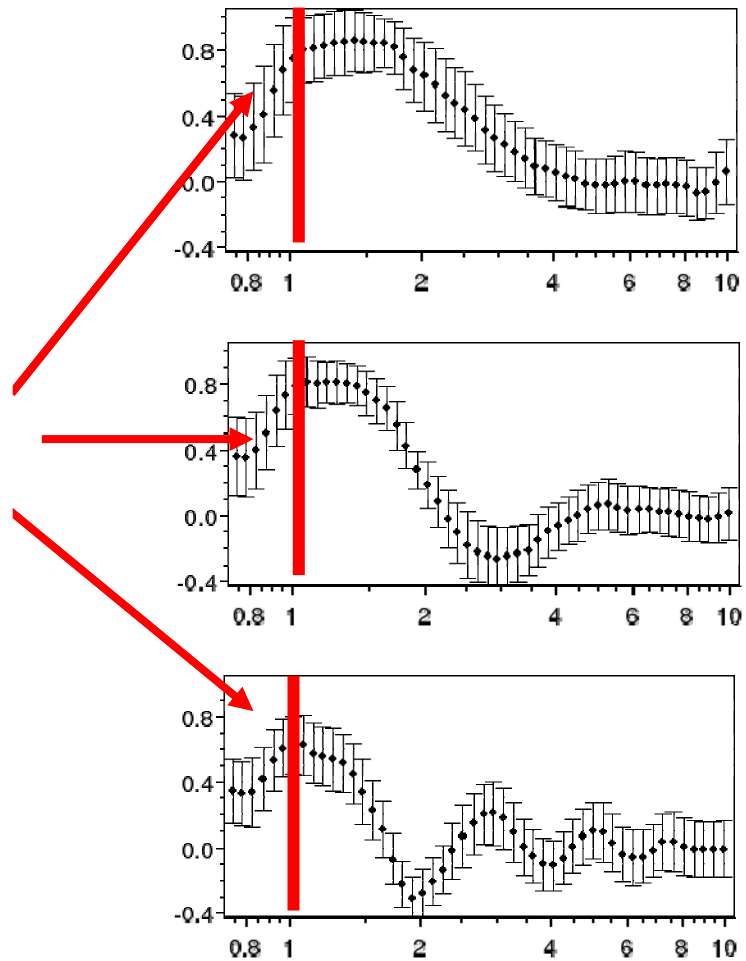


Spatial autocorrelation coefficient for different radius : resolution and non-uniqueness

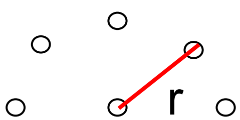
Theory

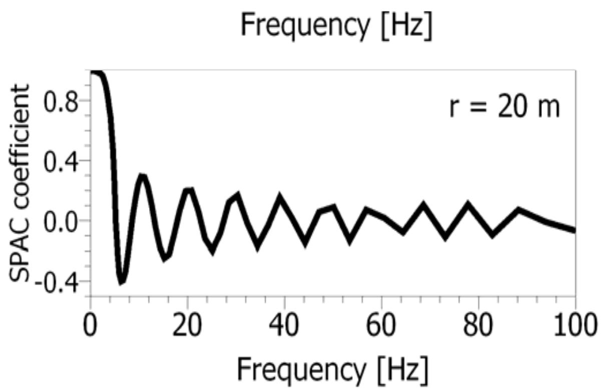
« real life »





Lack of energy in the signals

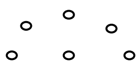


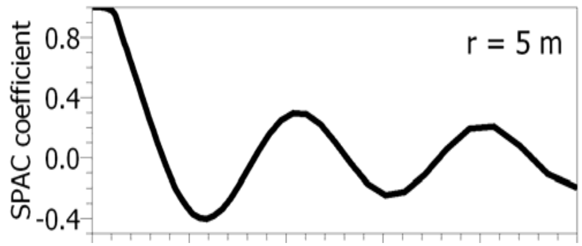


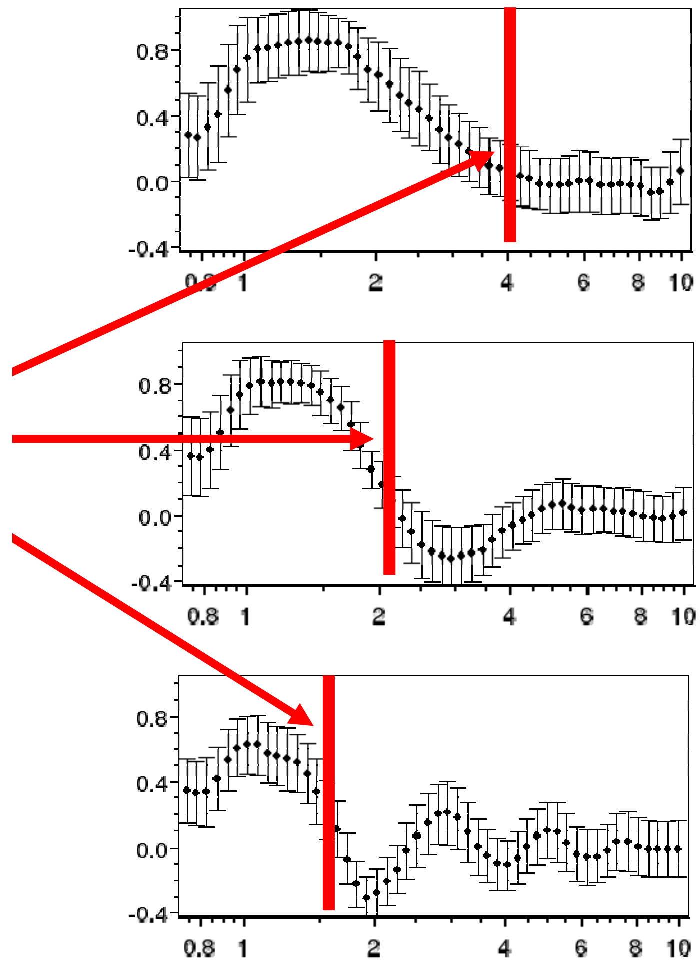
Spatial autocorrelation coefficient for different radius : resolution and non-uniqueness

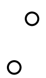
Theory

« real life »

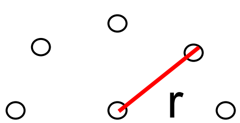


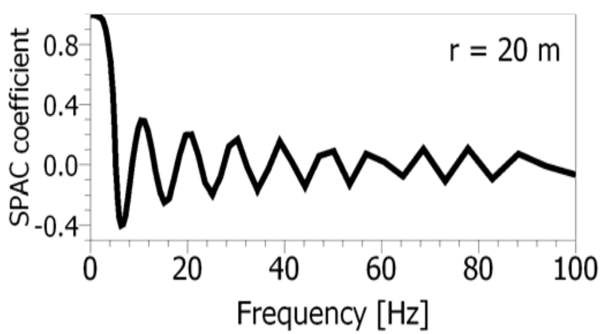






Non-uniqueness

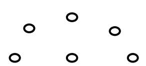


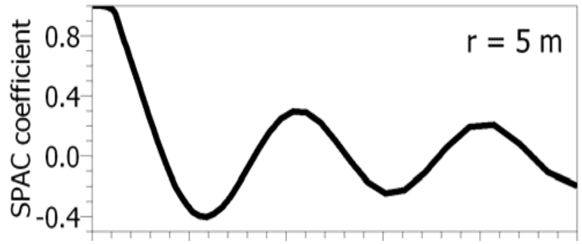


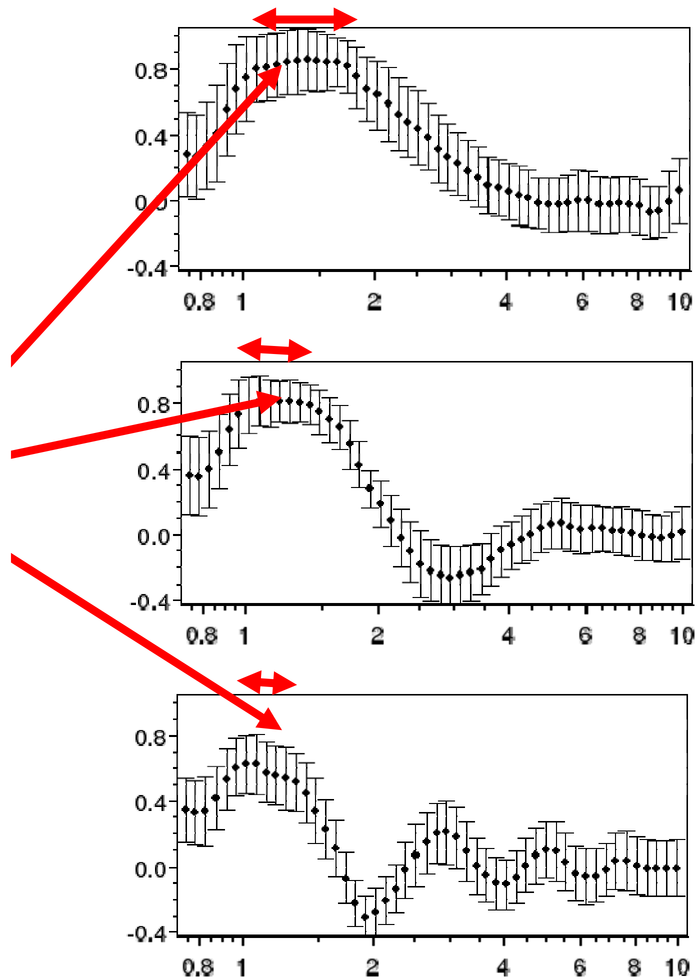
Spatial autocorrelation coefficient for different radius : resolution and non-uniqueness

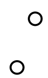
Theory

« real life »

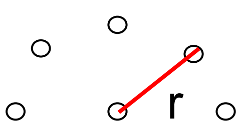


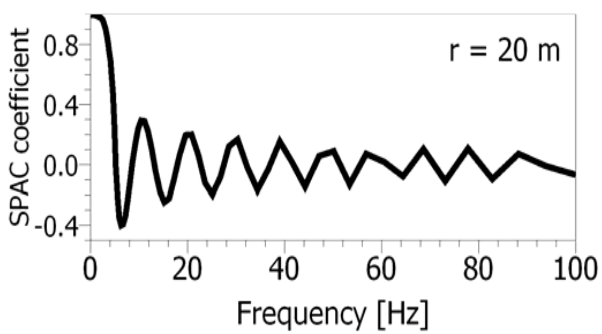






Lack of resolution

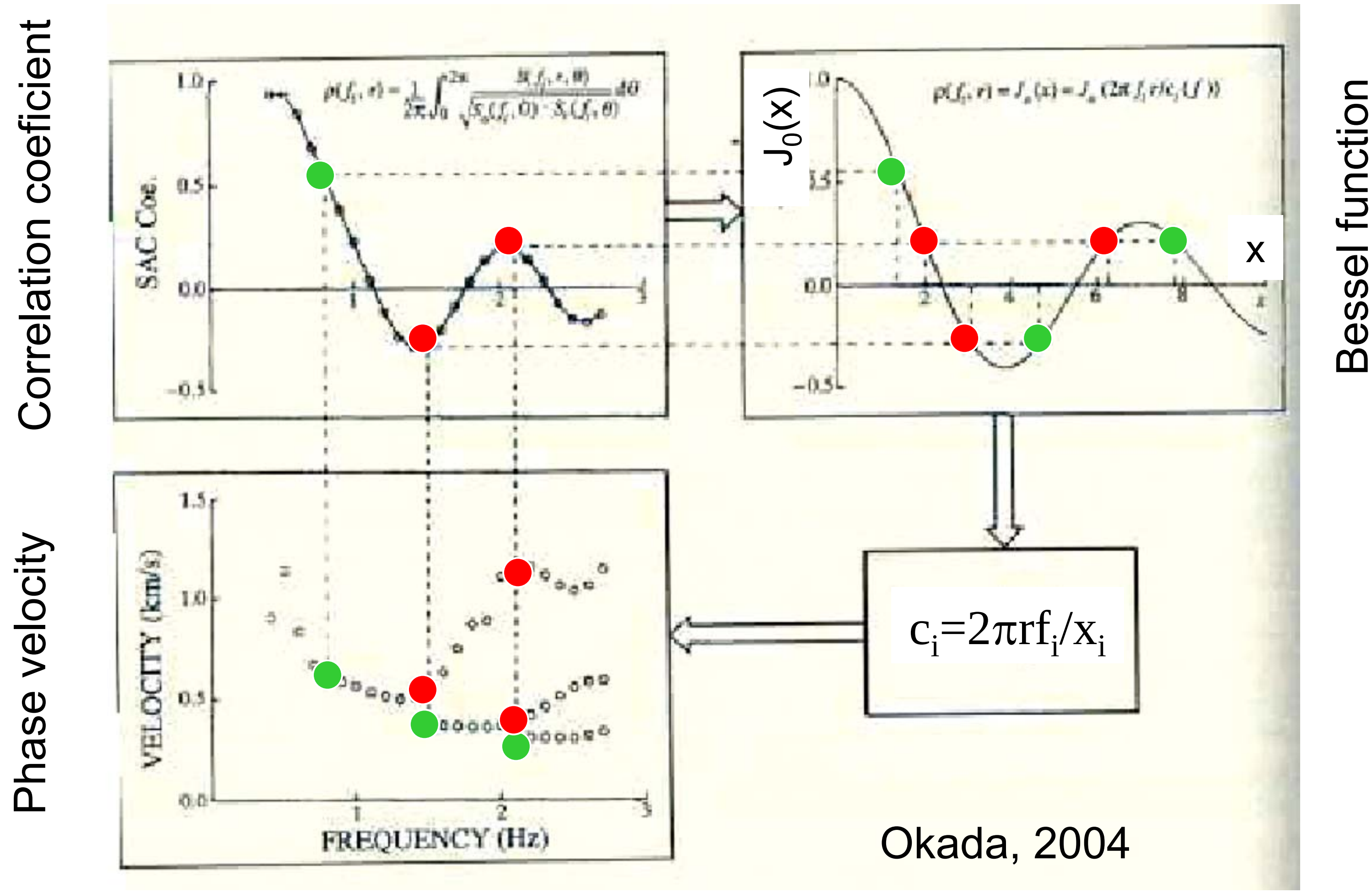




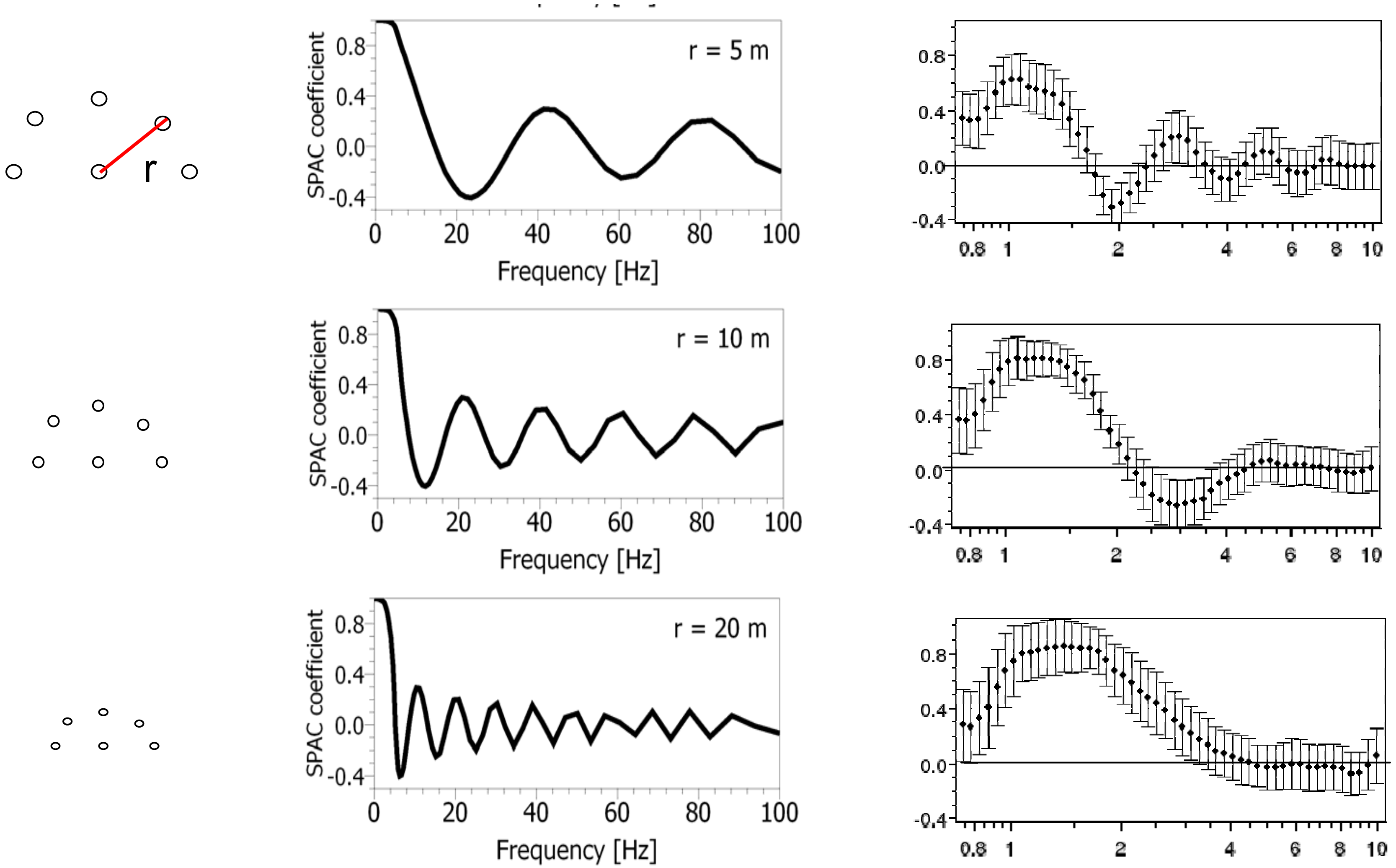
Estimation of phase velocity

$$\overline{\rho}(r,\omega) = J_0\left(\frac{\omega r}{c(\omega)}\right)$$

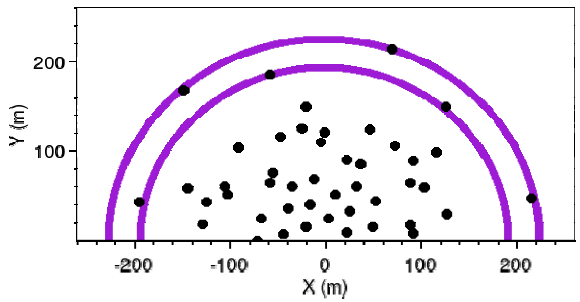
measure target

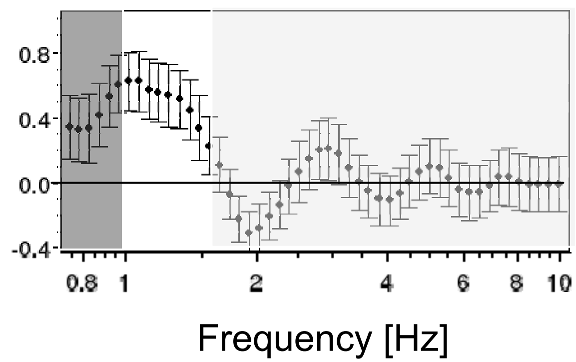


Spatial autocorrelation coefficient for different radius : resolution and non-uniqueness



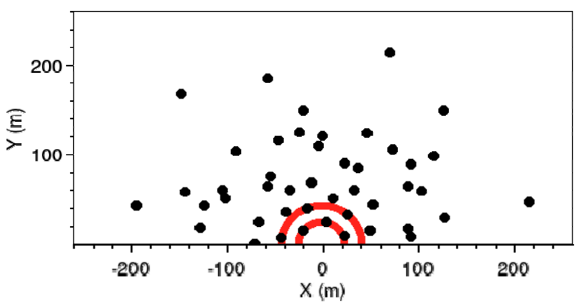
Array geometry: resolution and non-uniqueness

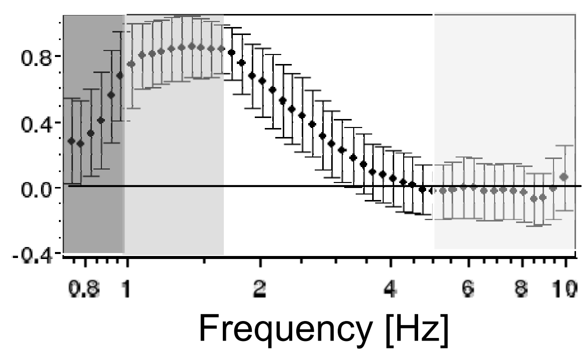




Largest ring controls the resolution

⇔ width of the f-k transfer function lobe

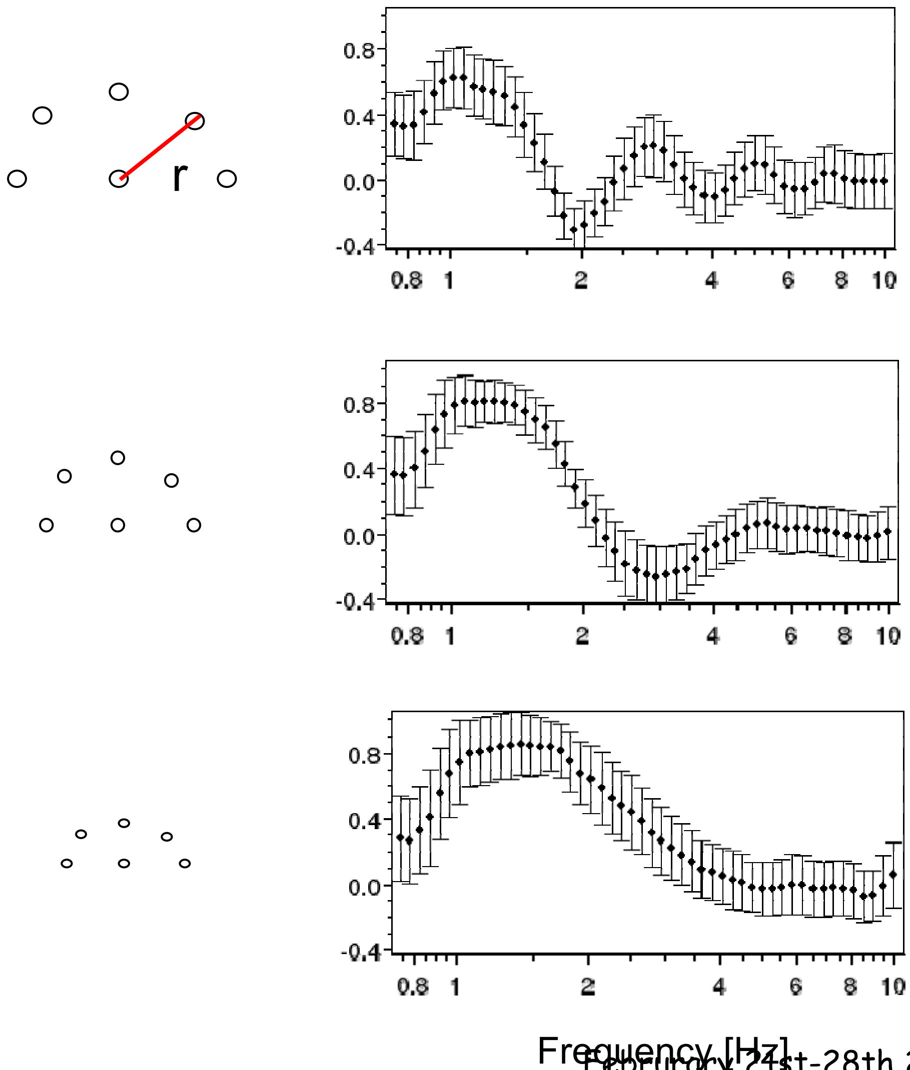


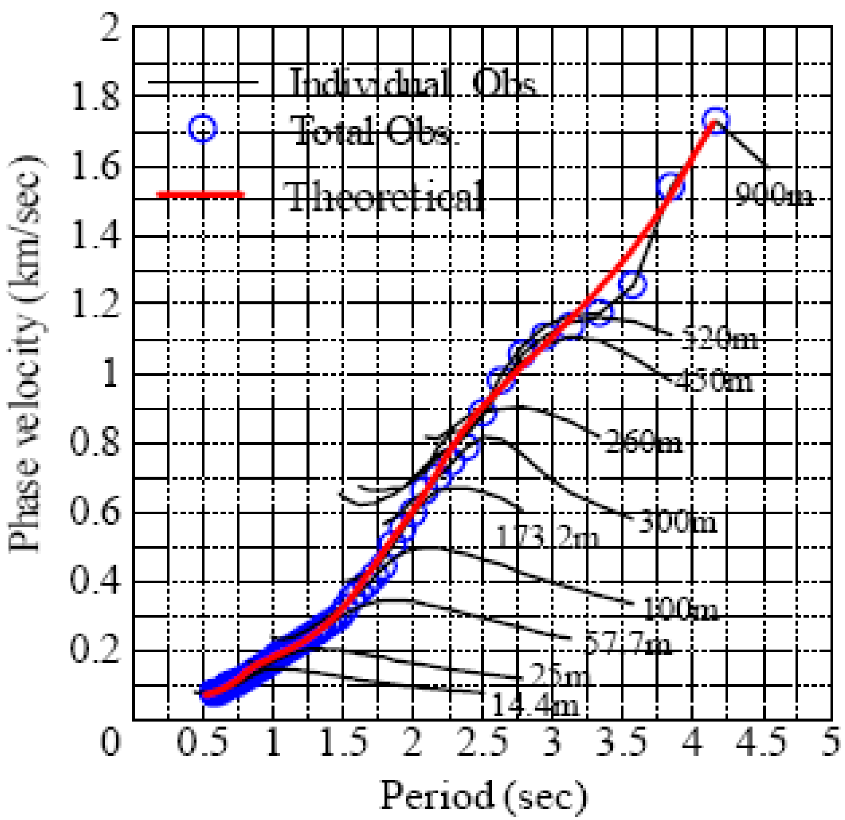


Smallest ring controls the non-uniqueness limit

⇔ location of f-k side lobes

Spatial autocorrelation coefficient for different radius : building a dispersion curve





Tsuno and Kudo, 2004

Trade-off between array aperture and smallest distance

-small distances/rings -> less non-uniqueness (first minimum at higher frequencies)

- large aperture/rings -> better resolution (plateau of Bessel functions)

Trade-off between thin rings and good station pair distribution

-Determination of inner and outer radius results from a compromise between the number of station pairs per ring (azimuthal resolution) and the ratio between ring thickness and ring radius

Finally, what are the main differences between FK and SPAC approaches ?

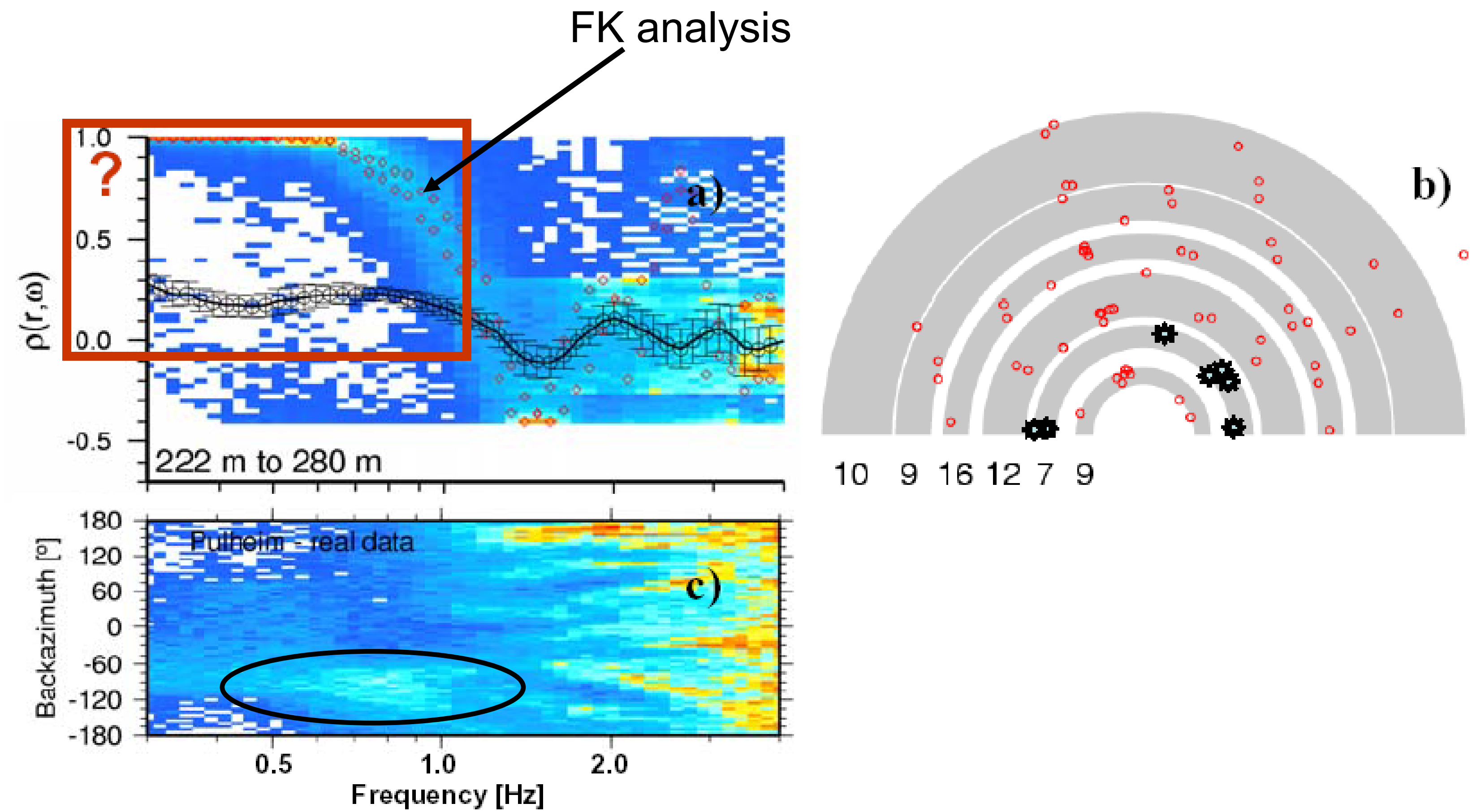
FK

- plane wave propagation
- one single source (or few uncorrelated sources)
- direct measure of phase delay

SPAC

- plane wave propagation
- large number of uncorrelated sources spatially and temporally randomly distributed
- measure of spatial (de-)coherency of waveforms

Effects of preferential sources direction on the average spatial autocorrelation coefficient



Ohrnberger (2004)

How to compute the autocorrelation coefficient?

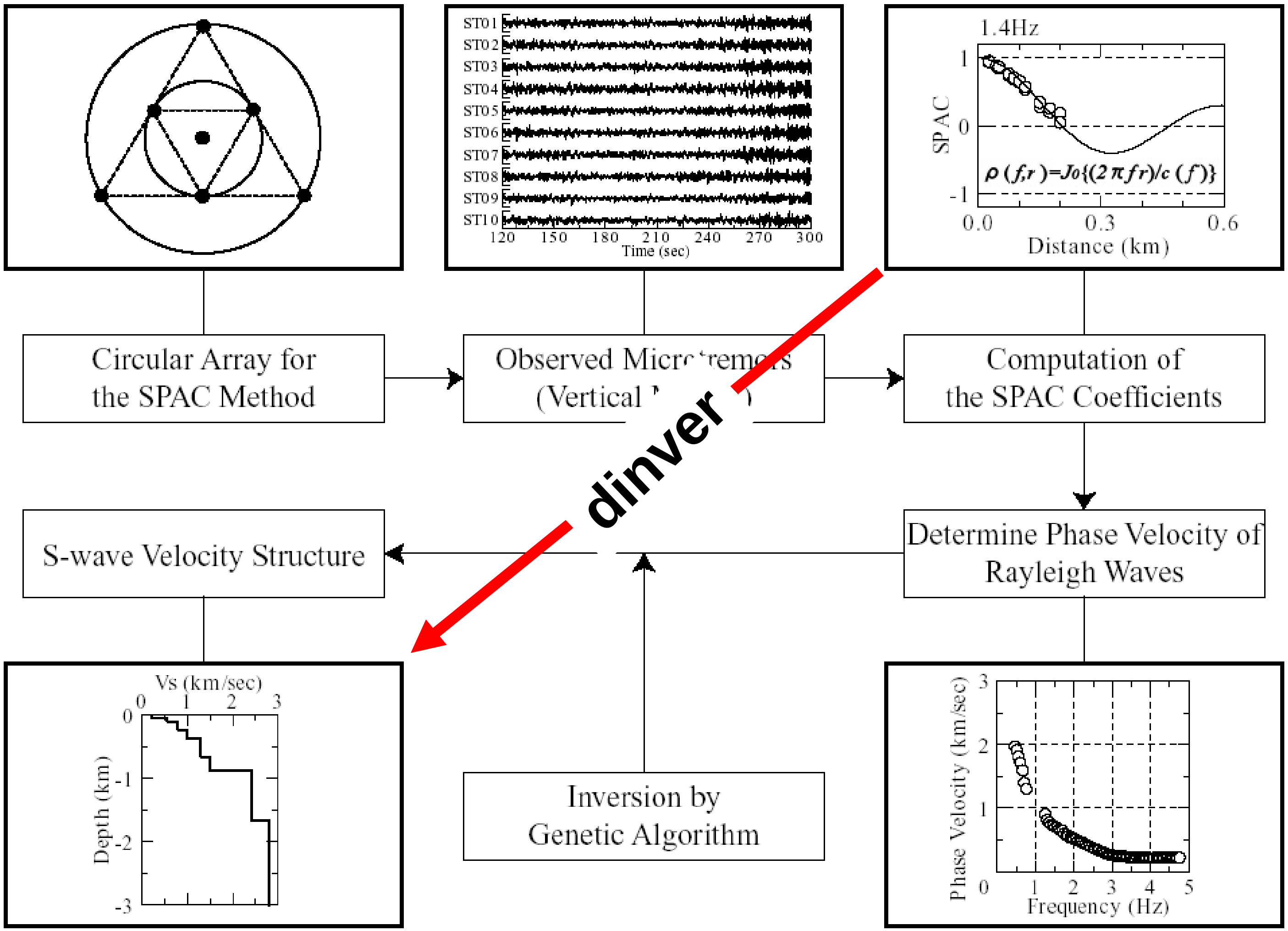
In the time domain (<=> sesarray)

- Compute the Fourier transform of signals
- Narrow band-pass filter around ω_0
- Inverse Fourier transform and computation of correlation
- Azimuthal averaging of the correlation coefficient

In the frequency domain

- Compute the cross-spectra normalized by the autospectra
- Azimuthal averaging on the real part of the cross-spectra

SPAC implementation



Kudo et al., 2002